Addition
THE MOST IMPORTANT NUMERACY STRATEGIES

Additive Reasoning includes reasoning about addition problems, like 48 + 29 as shown below. Additive Reasoning can be described as thinking in terms of bigger jumps of numbers than one by one counting. When reasoning additively, one considers the magnitudes of the numbers involved, thinking about 48 as 40 and 8, or almost 50, and 29 as 20 and 9, or almost 30. The strategies are identified by the student's plan of attack, the first move the student makes.

Splitting by Place-Value

This is the foundation strategy that many students will construct on their own, splitting each addend by place-value, adding those place-value parts together, and then pulling everything together. This is a necessary starting point, but then students need to develop the rest of these strategies.
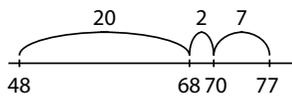
$$48 + 29$$

$$40 \quad 8 \quad 20 \quad 9$$

$$60 + 17 = 77$$

$48 + 29$
$= (40 + 8) + (20 + 9)$
$= (40 + 20) + (8 + 9)$
$= 60 + 17$
$= 77$

$48 = 40 + 8$
$+ 29 = 20 + 9$
$\quad\quad 60 + 17 = 77$

Add a Friendly Number

Keeping one addend whole, students decompose the other addend in order to add a friendly part of that addend. Then students continue to add the rest of that addend.

$48 + 29$
$= 48 + (20 + 9)$
$= (48 + 20) + 9$
$= 68 + 9$
$= 68 + (2 + 7)$
$= (68 + 2) + 7$
$= 70 + 7$
$= 77$
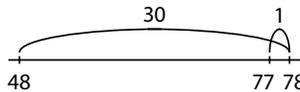
Get to a Friendly Number

Keeping one addend whole, students decompose the other addend in order to add to a friendly number. Then students continue to add the rest of that addend. This is a bit sequential in nature - get to the friendly number, take stock, add the rest.
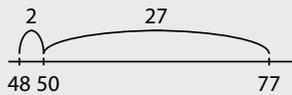
$48 + 29$
$= 48 + (2 + 27)$
$= (48 + 2) + 27$
$= 50 + 27$
$= 77$

Add a Friendly Number - Over

This strategy is a version of Add a Friendly Number where students adds a friendly number that is too big and then must adjust back.

$48 + 29$
$= 48 + (30 - 1)$
$= (48 + 30) - 1$
$= 78 - 1$
$= 77$

Give and Take (Compensation)

This strategy is a more sophisticated version of the Get to a Friendly Number strategy because rather than being sequential, the student gives and takes almost simultaneously. "If I take that from one addend to give to the other addend, I can create an equivalent problem that is easier to solve."

$$\begin{array}{c} 48 \;+\; 29 \\ +2 \;\; -2 \\ \hline 50 + 27 \;=\; 77 \end{array}$$

$$\begin{array}{r} 48 \;+2 = 50 \\ + 29 \;-2 = 27 \\ \hline 77 \end{array}$$
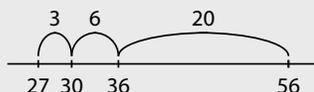
Subtraction
THE MOST IMPORTANT NUMERACY STRATEGIES

Additive Reasoning includes reasoning about subtraction problems, like 56 − 29 as shown below. Additive Reasoning can be described as thinking in terms of bigger jumps of numbers than one by one counting. The strategies are identified by the student's plan of attack, the first move the student makes. Subtraction can be thought of as both removal and difference/distance. Both interpretations are important for subtracting numbers but also for many applications in higher math, including integer operations.
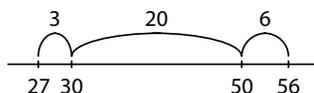
Remove a Friendly Number

Keeping the minuend whole, students decompose the subtrahend in order to subtract a friendly amount of the subtrahend. Then students continue to subtract the rest of the subtrahend.

$56 - 29$
$= 56 - 20 - 9$
$= 36 - 9$
$= 36 - 6 - 3$
$= 30 - 3$
$= 27$

Remove to a Friendly Number

Keeping the minuend whole, students decompose the subtrahend in order to remove to a friendly number. Then students continue to remove the rest of the subtrahend.

$56 - 29$
$= 56 - 6 - 23$
$= 50 - 23$
$= 50 - 20 - 3$
$= 30 - 3$
$= 27$

Remove a Friendly Number - Over

This strategy is a version of Remove a Friendly Number where students remove a number that is too big and then must adjust back.
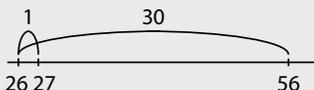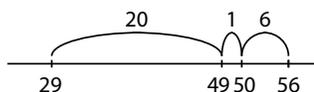
$56 - 29$
$= 56 - (30 - 1)$
$= (56 - 30) + 1$
$= 26 + 1$
$= 27$

Find the Distance (Difference)

This strategy makes use of the relationship between addition and subtraction, where students find the missing addend in the related addition problem. This is equivalent to finding the distance between the subtrahend and the minuend, ideally in friendly jumps.
Symbolically: c − a = __ if and only if a + __ = c

$29 + 1 = 30$
$30 + 20 = 50$
$50 + 6 = 56$
$1 + 20 + 6 = 27$

Constant Difference (Equivalent Difference)

In this most sophisticated strategy, students consider the difference between the subtrahend and minuend (instead of removal). Students then create an equivalent problem that's easier to solve by shifting the difference by adding to or subtracting from both subtrahend and minuend.

$$\begin{array}{r} 56 \;-\; 29 \\ +1 \quad +1 \\ \hline 57 \;-\; 30 = 27 \end{array}$$

$$\begin{array}{r} 56 \;+1 = \quad 57 \\ -\;29 \;+1 = -30 \\ \hline 27 \end{array}$$