

Okay, so my one gripe with CAMI is that the problems posed have literally caused me to lose sleep more than once. In my bed last night, my head just about exploded.

After our meeting, I had a 1-hour train ride home. On the train, I recreated one of the addition tables we had looked at at the meeting to help answer the question how many total possible ways could $? + ? = n$, where $n =$ a whole number, we can only use the digits 1-9, and only once. However, after drawing it the way we had it at the meeting I thought, what if I organized the table to look more like x and y -axes?

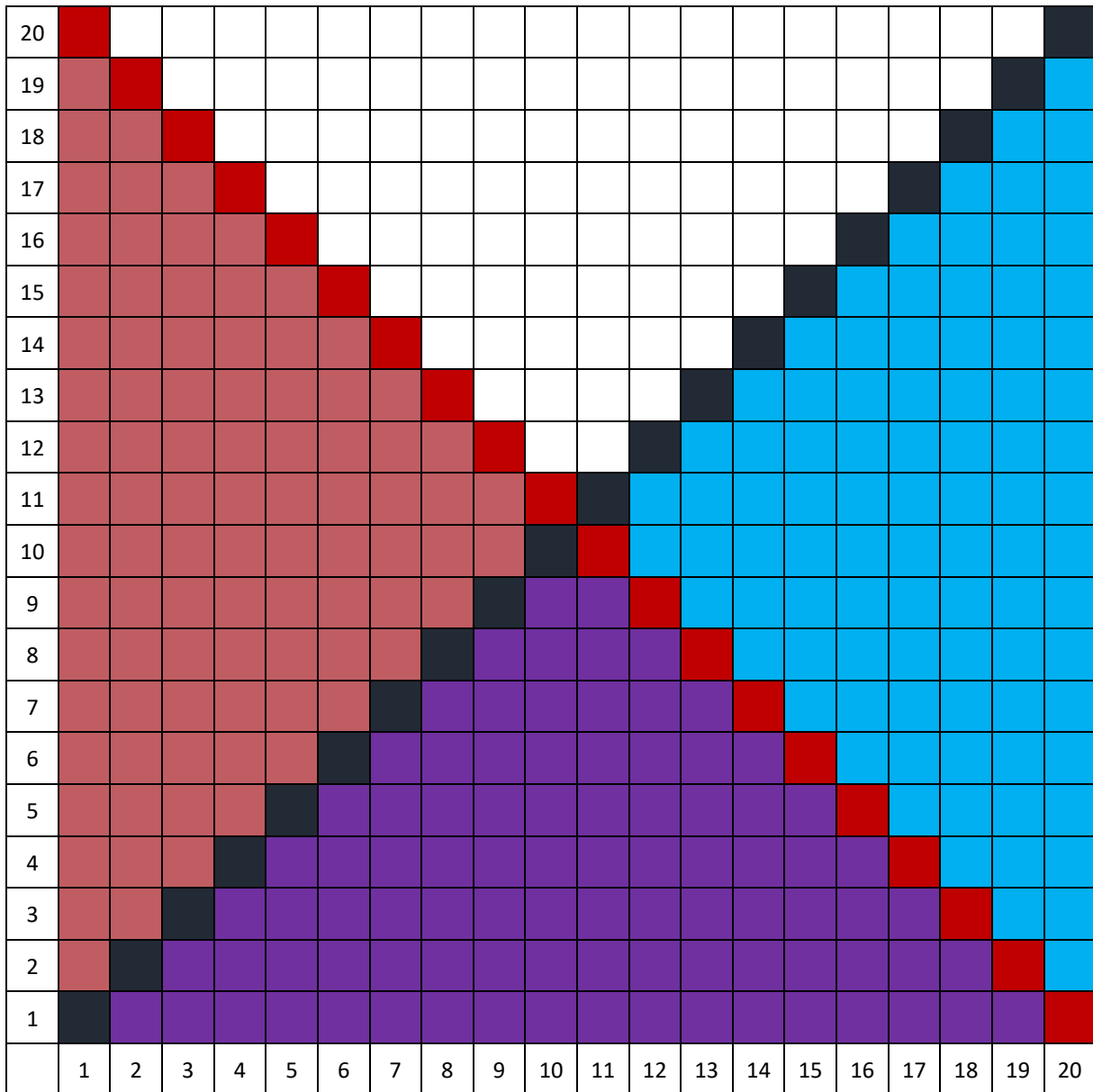
For those rules, if $n = 8$, then we could come up with the following graph. We can change the $? + ? = n$, to $x + y = n$. For example, if $n = 8$, we have the following:

8								
7								
6								
5								
4								
3								
2								
1								
	1	2	3	4	5	6	7	8

Adding the purple boxes (solutions) in the rows we have $6 + 4 + 2 = 12$ possible ways. An example of a solution, would be (3,1) because $3 + 1 = 4$ and $4 \leq 8$.

Immediately, I thought, that image reminds me of systems of inequalities! But, before I went down that path, I made bigger graphs to help me see a larger example to help look at the pattern more easily. But, of course, this violates the 1-9 only rule.

$$n = 20$$



$$\text{Total Outcomes} = 18 + 16 + 14 + 12 + 10 + 8 + 6 + 4 + 2 = 10^2 - 10 = 90$$

$$\frac{n}{2} \left(\frac{n}{2} - 1 \right)$$

One of the cool things here, to me, is that each color represents something:

Purple = solution set (Goldilocks zone)

Pink = duplicates

Light blue = sum is too big

White = sum is too big and duplicate

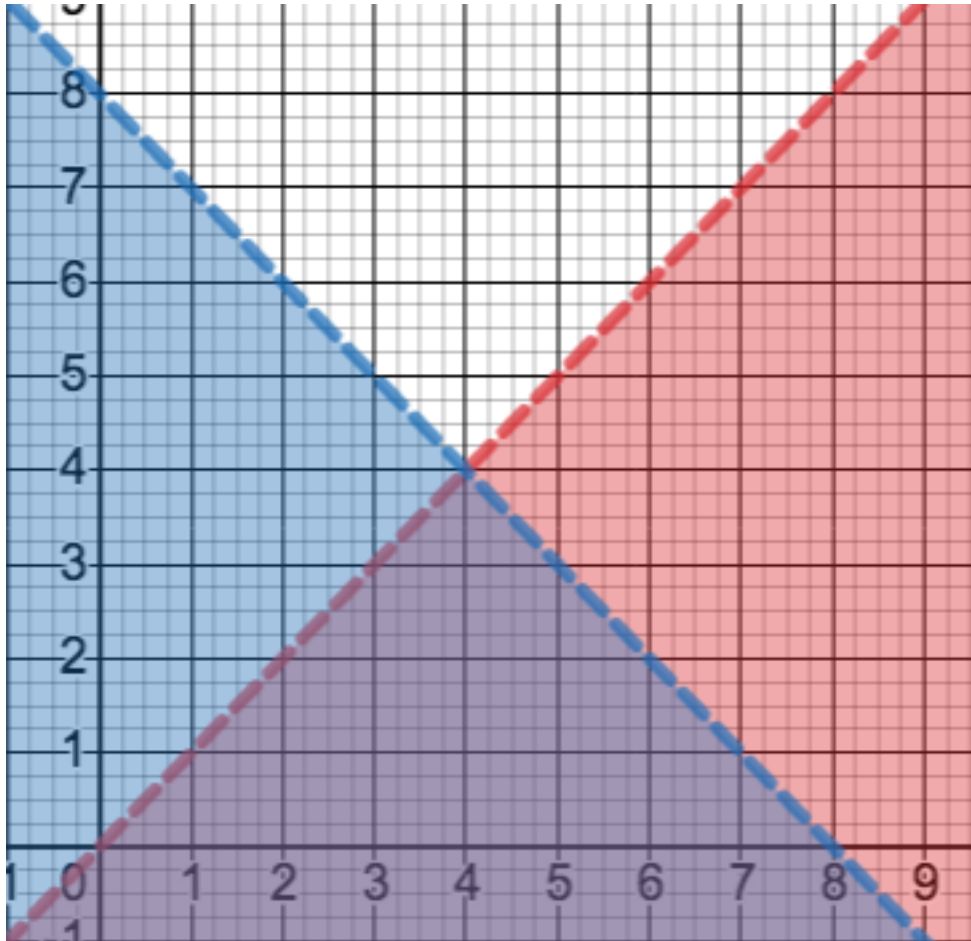
Deep, dark blue = same number twice

Red = ? = boundary line for too big? (maybe this line isn't necessary, but it makes it symmetrical)

Then, I got to thinking. What if I tried to graph this as systems of inequalities, but forget the whole whole number thing? What if we allowed all numbers up to n ? For example, the graph when $n = 8$, can be done with:

$$y < x$$

$$y < -x + 8$$



And, if we want to see how the graph changes when n changes, we can use Desmos to graph:

$$y < x$$

$$y < -x + n$$

using their “slider” feature.

Of course, when answering the question, how many solutions, the answer here is infinite. But, we can place a size on these “infinite” areas? For example, if $n = 8$, the solution set is found in the area $\frac{1}{2}(4)(8) = 16$ OR 4^2 .

If $n = 9$, we get $\frac{1}{2}(4.5)(9)$ OR $(4.5)^2 = 20.25$

If $n = 10$ we get $\frac{1}{2}(5)(10) = 5^2 = 25$. Eric, those are your squares!

Now, there is more than one rabbit hole I can jump down from here. Allowing for all numbers eliminates the whole odd vs even problem experienced by the whole number graphs. Then we don't need two separate functions. Then, I started to think about how whole numbers are just an illusion created by our number system. Then to thinking about how creating functions to explain geometric models using squares is a process of reverse engineering math to account for the piecewise functions created by the whole number phenomenon. But, alas, I am back to work, and the rabbit holes will have to wait for another time.

Well, that is one train-ride home, one-train ride back to work, and laying in bed sleeplessly thinking about this. Sorry for the rambling, and thank you Ramon for the headache. I mean that in the best possible way.

-Todd Orelli