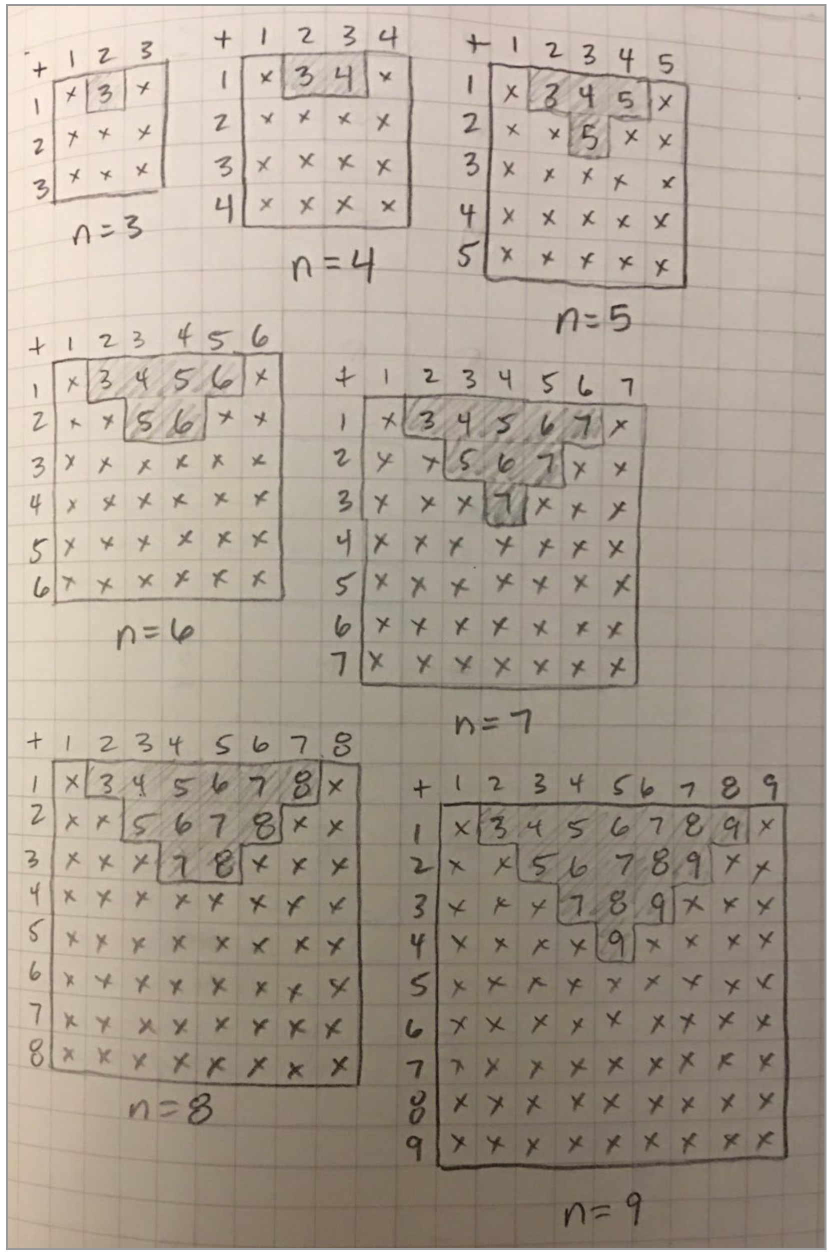
This is inspired by Todd's response after Ramon's meeting on: _ + _ = _

Todd's use of addition tables to create solutions got me thinking about a visual pattern based on triangles and squares.

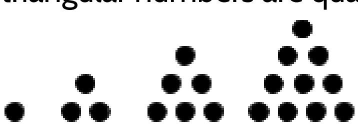
The addition tables show solutions to _ + _ = _ for different number of possible digits (n). For example, when we are allowed only the first 3 digits (1, 2, 3), 1 + 2 = 3 is the only solution. When we are allowed only the first 4 digits (1, 2, 3, 4), 1 + 2 = 3 and 1 + 3 = 4 are only two solutions.

Here are the solutions shown:

n	f(n)
3	1
4	2
5	4
6	6
7	9
8	12
9	16

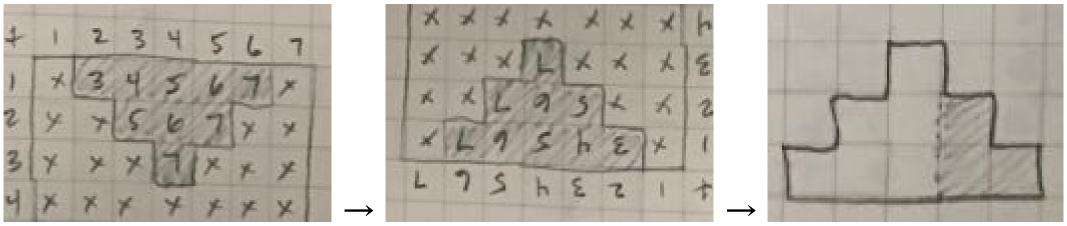


I noticed a couple things when looking at the numbers here. The solutions for odd n's are square numbers: 1, 4, 9, 16. I also saw triangular numbers in odd and even n's. As a reminder, triangular numbers are quantities that can be arranged as triangles:

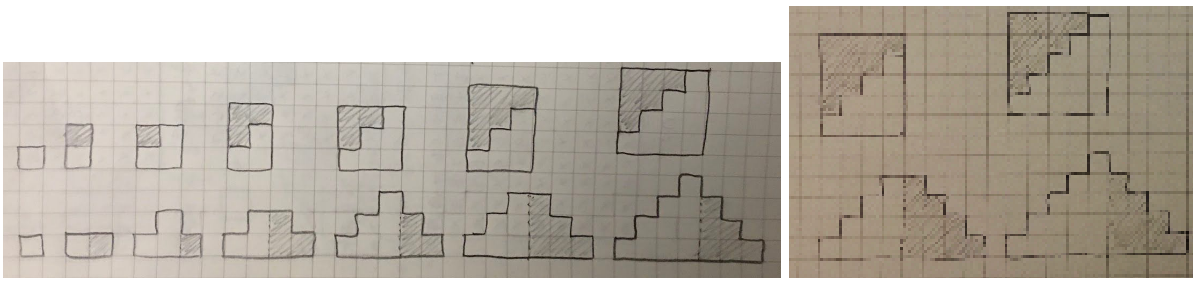
 (1, 3, 6, 10, ...).

The solutions for even n's are 2 times the triangular numbers: 2(**1**), 2(**3**), 2(**6**), etc.

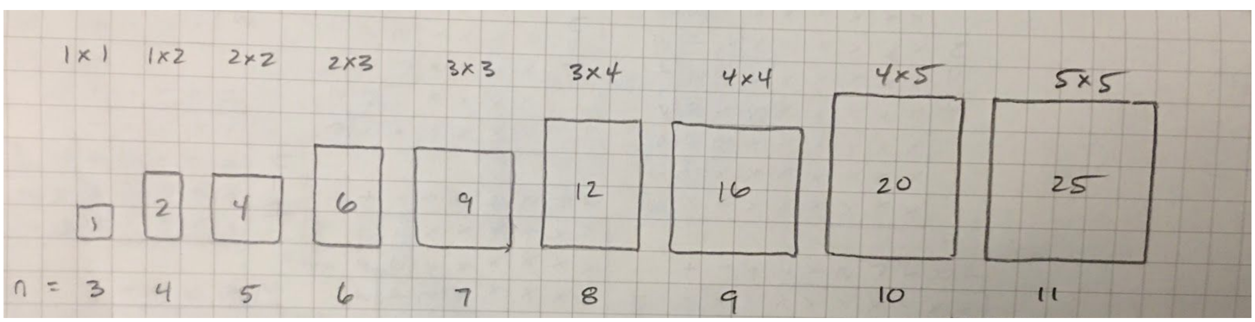
I wondered about the shape of solutions in the addition tables. I saw pyramids, which I flipped over in order to see a visual pattern. For example, these are the solutions for n=7:



The bottom row shows the shape of the solutions, flipped upside down. The top row shows the same solutions, reshuffled to make a square or a (non-square) rectangle. Both the squares and the rectangles are made of triangles.



Here is the top row again, with the number of solutions shown as well as the dimensions of the figure: 1x1, 1x2, 2x2, 2x3, 3x3, etc.



When I look at this visual pattern, I see the dimensions growing by 1 unit, alternating up and across as n grows by 1. So, the 1x1 square grows horizontally by 1 to become 1x2, then grows vertically by 1 to become 2x2, then grows horizontally by 1 to become 2x3, then vertically... it goes up, then across, then up, then across.

By the way, the number of solutions for even n's (1x2, 2x3, 3x4, 4x5, 5x6, etc.) are called rectangular, oblong, or pronic numbers. I have no idea why there are three different names.

The visual pattern made really want to know what function would produce this series of numbers. I also wondered if the series 1, 2,4, 6, 9, 12, 16, 20 shows up other places, so I searched the Online Encyclopedia of Integer Sequences[1] (http://oeis.org/) for the sequence. It turns out that this series is called the quarter-squares and they have a rich history going back to the Babylonians and have been used recently in computer programming because of a cool property they have.

They are called quarter-squares because to produce them you square an integer then divide it by 4. If there is a non-integer part, throw it away. Here's a table showing how the series is calculated:

n	square it	divide by 4	throw away decimal part (if any)
1	1	.25	0
2	4	1	1
3	9	2.25	2
4	16	4	4
5	25	6.25	6
6	36	9	9
7	49	12.25	12
8	64	16	16
9	81	20.25	20
10	100	10	10
n	n^2	$\frac{n^2}{4}$	$floor(\frac{n^2}{4})$

I'm not going to try to explain it, but I also found out the quarter-squares have been used to simplify multiplication before modern computers.

Thank you, Ramon! What a great exploration.

[1] If you haven't been to OEIS before, check it out. You might look up triangular numbers, pronic numbers, etc.