Congressional Apportionment

**Hamilton Method**

**Exercise 1**: For the 3 states given, apportion a number of congressional seats N, with **N=37**.

1. Determine the **divisor** by dividing the total country population by the number of representatives. $\frac{total population}{seats}$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_ (keep at least 2 decimal places)
2. Calculate the “**quota**” for each state by dividing the state’s population by the **divisor**. Keep at least 2 decimal places.
3. Separate the **integer part** and the **decimal part** of the quota. The integer part is the initial number of seats given to this state.
4. Total the integer parts of the quota. If the total is the same as the number of total seats available, you’re done.
5. Additional seats to apportion are given to states with the highest **decimal part**, one at a time until they are gone.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| State | Pop. | Quota | Integer part | Decimal part | Additional Seats | Final # of Seats |
| A | 1500 |  |  |  |  |  |
| B | 1000 |  |  |  |  |  |
| C | 100 |  |  |  |  |  |
| Total: |  | Total: |  |  |  |  |

**Exercise 2**: For the 3 states given, apportion a number of congressional seats N, with **N=38**.

$Divisor\_{} =\frac{total population}{seats}$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| State | Pop. | Quota | Integer part | Decimal part | Additional Seats | Final # of Seats |
| A | 1500 |  |  |  |  |  |
| B | 1000 |  |  |  |  |  |
| C | 100 |  |  |  |  |  |
| Total: |  | Total: |  |  |  |  |

What do you notice when you compare the results?

**Exercise 3**: For the 3 states given, apportion a number of congressional seats N, with **N=100**, first in the year 2000 and then in the year 2010.

$Divisor\_{} =$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| State | Pop.(2000) | Quota | Integer part | Decimal part | Additional Seats | Final # of Seats |
| D | 746 |  |  |  |  |  |
| E | 2217 |  |  |  |  |  |
| F | 7037 |  |  |  |  |  |
| Total: |  | Total: |  |  |  |  |

$Divisor\_{} =$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| State | Pop.(2010) | Quota | Integer part | Decimal part | Additional Seats | Final # of Seats |
| D | 750 |  |  |  |  |  |
| E | 2128 |  |  |  |  |  |
| F | 6960 |  |  |  |  |  |
| Total: |  | Total: |  |  |  |  |

What do you notice when you compare the results?

**Jefferson Method**

The first steps of Jefferson’s method are the same as Hamilton’s method. He finds the same divisor and the same quota, and cuts off the decimal parts, giving a total number of representatives that is usually less than the required total. The difference is in how Jefferson resolves that difference. He says that since we ended up with an answer that is too small, our divisor must have been too big. He changes the divisor by making it smaller, finding new quotas with the new divisor, cutting off the decimal parts, and looking at the new total, until he finds a divisor that produces the required total.

**Exercise 4**: Using the Jefferson Method, apportion **76** seats to the following states.

$Divisor\_{} =$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_ $Adjusted Divisor\_{} =$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| State | Pop. | Quota | QuotaRounded Down | Adjusted Quota | Adjusted Quota Rounded Down |
| A | 107658 |  |  |  |  |
| B | 27744 |  |  |  |  |
| C | 25170 |  |  |  |  |
| D | 19951 |  |  |  |  |
| E | 14610 |  |  |  |  |
| F | 9225 |  |  |  |  |
| G | 3292 |  |  |  |  |
| Total: |  | Total: |  | Total: |  |

If you haven’t apportioned enough seats, you adjust **divisor** by…

If you have apportioned too many seats, you adjust **divisor** by…

Compare the results above to one of the other methods.

**Adams Method**

Adams finds a divisor and quota like Hamilton did, but instead of cutting off the decimal parts, he rounds up, giving a total number of representatives that is usually more than the required total. Adams says that since we ended up with an answer that is too big, our divisor must have been too small. He changes the divisor by making it bigger, finding new quotas with the new divisor, cutting off the decimal parts, and looking at the new total, until he finds a divisor that produces the required total.

**Exercise 5**: Using the Adams Method, apportion **76** seats to the following states.

$Divisor\_{} =$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_ $Adjusted Divisor\_{} =$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| State | Pop. | Quota | QuotaRounded Up | Adjusted Quota | Adjusted Quota Rounded Up |
| A | 107658 |  |  |  |  |
| B | 27744 |  |  |  |  |
| C | 25170 |  |  |  |  |
| D | 19951 |  |  |  |  |
| E | 14610 |  |  |  |  |
| F | 9225 |  |  |  |  |
| G | 3292 |  |  |  |  |
| Total: |  | Total: |  | Total: |  |

If you haven’t apportioned enough seats, you adjust **divisor** by…

If you have apportioned too many seats, you adjust **divisor** by…

Compare the results above to one of the other methods.

**Webster Method**

Webster finds a divisor and quota like Hamilton did, but instead of cutting off the decimal parts, he rounds up or down by normal rounding rules. If the total number of representatives is more than the required total, he changes the divisor by making it bigger, finding new quotas with the new divisor. If the total number of representatives is less than the required total, he changes the divisor by making it smaller, finding new quotas with the new divisor. He repeats this process until he finds a divisor that produces the required total.

**Exercise 6**: Using the Webster Method, apportion **76** seats to the following states.

$Divisor\_{} =$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_ $Adjusted Divisor\_{} =$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| State | Pop. | Quota | QuotaRounded | Adjusted Quota | Adjusted Quota Rounded |
| A | 107658 |  |  |  |  |
| B | 27744 |  |  |  |  |
| C | 25170 |  |  |  |  |
| D | 19951 |  |  |  |  |
| E | 14610 |  |  |  |  |
| F | 9225 |  |  |  |  |
| G | 3292 |  |  |  |  |
| Total: |  | Total: |  | Total: |  |

If you haven’t apportioned enough seats, you adjust **divisor** by…

If you have apportioned too many seats, you adjust **divisor** by…

Compare the results above to one of the other methods.

**Huntington-Hill Method**

This is the method that is currently used for the House of Representatives.

1. Determine how many people each representative should represent. Do this by dividing the total population of all the states by the total number of representatives to find the **divisor**.
2. Divide each state’s population by the divisor to determine how many representatives it should have. Record this answer to several decimal places. This answer is called the **quota**.
3. Cut off the decimal part of the quota to obtain the **integer part** of the quota, which we will call $n$. Compute $\sqrt{n(n +1)}$, which is the **geometric mean** of the integer part of the quota and one value higher.
4. If the quota is larger than the geometric mean, round up the quota; if the quota is smaller than the geometric mean, round down the quota. Add up the resulting whole numbers to get the **total initial allocation**.
5. If the total from Step 4 was less than the total number of representatives, reduce the divisor and recalculate an **adjusted quota** and total allocation, using the geometric mean to round. If the total from step 4 was larger than the total number of representatives, increase the divisor. Continue doing this until the total is equal to the total number of representatives. The divisor we end up using is called the **adjusted divisor**.

**Exercise 7**: Using the Huntington-Hill Method, apportion **76** seats to the following states.

$Divisor\_{} =$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_ $Adjusted Divisor\_{} =$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| State | Pop. | Quota | Integer Part | Decimal Part | Geometric Mean | Initial Allocation | Adjusted Quota | Final Number of Seats |
| A | 107658 |  |  |  |  |  |  |  |
| B | 27744 |  |  |  |  |  |  |  |
| C | 25170 |  |  |  |  |  |  |  |
| D | 19951 |  |  |  |  |  |  |  |
| E | 14610 |  |  |  |  |  |  |  |
| F | 9225 |  |  |  |  |  |  |  |
| G | 3292 |  |  |  |  |  |  |  |
| Total: |  | Total: |  |  | Total: |  | Total: |  |