Research Findings and Essentials of Statistics[1]

High school graduates need to be able to make sense of the storm of data that is continuously in front of them. They will encounter data with variation and error in polls, in the news, at the doctor's office, assessing themselves and in the workplace. They will also need to be posing questions and seeking reliable results.

Definition of Statistics
Statistics is the science of learning from data, and of measuring, controlling, and communicating uncertainty (Davidian, M. and Louis, T. A., 10.1126/science.1218685 & amstat.org). People study statistics for a variety of reasons, including (?)[2]:

1. To understand the statistical studies performed in their field (i.e., be knowledgeable about the vocabulary, symbols, concepts, and statistical procedures used in those studies).
2. To conduct research in their field (i.e., be able to design experiments and samples; collect, organize, analyze, and summarize data; make reliable predictions or forecasts for future use; and communicate statistical results).
3. To be better consumers of statistical information.

Statistical literacy is recognized as essential in contributing to the success of dealing with the requirements of citizenship, employment, continuing education, family and to be prepared for a healthy happy , and productive life (Franklin et al. 2007; College Board 2006; American Diploma Project, 2004).

Statistical literacy:

- enables interpretation of data and promotes thoughtful questioning;
- enables sound choices about politics, polls, nutrition, health and medicine, and product selection;
- provides workplace skills for assessing quality and accountability as wells as understandings for how to assess the tools and errors in assessment data; and
- enables an important skepticism toward what gets labeled as scientific findings (Garfield & Ben-Zvi, 2007).

Statistical literacy and reasoning involve understanding and using the basic language and tools of statistics such as distribution, center and variation and making sense of statistical results.

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[2] http://derekogle.com/NCMTH107/book/Why%20Statistics%20is%20Important.pdf

Teaching Statistics

The statistical problem solving process can be described with the following four concepts (GAISE):
I. Formulating Questions: Anticipating variability and establishing an appropriate statistics question;
II. Collecting Data: Acknowledging variability and designing for differences;
III. Analyzing Data: Selecting appropriate graphical and numerical methods and using these methods to analyze the data; and
IV. Interpreting Results: Using the data and its variability to interpret results and interpreting results beyond the data set to answer the original questions.

The modules selected and adapted are intended to serve high school students who have not yet completed a course focused on *statistics with probability*. While greater proficiency requires more study over a longer period of time, these materials are intended to provide about a quarter of a yearlong high school math course. More time could certainly be invested in the topics presented here and on more advanced statistical topics.

RESEARCH
Research suggests that it takes time, well-ordered lessons activities and discussion questions, revisited topics, and appropriate technological tools to develop deep understanding of statistics (Garfield & Ben-Zvi, 2007). Teachers need to be aware of common statistical misconceptions, and research supporting successful teaching of statistics.

Misconceptions
Some misconceptions in statistics may be linked with poor understandings in proportional reasoning which is fundamental to probability and statistics. Some statistical and probabilistic truths contradict student intuitions and these are resilient and difficult to change (Konold, 1989b; Well et al., 1990; delMas & Garfield, 1991). Source (Garfield, 1995). Instructors cannot expect students to ignore their strong intuitions merely because they are given contradictory information in class. Common examples of these misconceptions are provided below (Garfield, 1995):

	Example of misconception	General Description
Representativeness	• If you are randomly sampling sequences of 6 births in a hospital, where B represents a male birth and G a female birth; BGGBGG is believed to be a more likely outcome than BBBBBG • 70% Heads is believed to be just as likely an outcome for 1000 tosses as for 10 tosses of a fair coin.	People estimate the likelihood of a sample based on how closely it resembles the population. Use of this heuristic also leads people to judge small samples to be as likely as large ones to represent the same population.
Gambler's fallacy (aka Monte Carlo's fallacy)	After observing a long run of heads, most people believe that now a tail is 'due' because the occurrence of a tail will result in a more representative sequence than the occurrence of another head. (Garfield, 1995)	Use of the representative heuristic leads to the view that chance is a self-correcting process.

Base-rate fallacy	A group of policemen have breathalyzers displaying false drunkenness in 5% of the cases in which the driver is sober. However, the breathalyzers never fail to detect a truly drunk person. 1/1000 of drivers are driving drunk. Suppose the policemen then stop a driver at random, and force the driver to take a breathalyzer test. It indicates that the driver is drunk. We assume you don't know anything else about him or her. How high is the probability he or she really is drunk? Many would answer as high as 0.95, but the correct probability is about 0.02.	People ignore the relative sizes of population subgroups when judging the likelihood of contingent events involving the subgroups.
Availability bias	Estimating the divorce rate in your community by recalling the divorces of people you know, or estimating the risk of a heart attack among middle-aged people by counting the number of middle-aged acquaintances who have had heart attacks.	Strength of association is used as a basis for judging how likely an event will occur. As a result, people's probability estimates for an event are based on how easily examples of that event are recalled. (Garfield, 1995)
Conjunction fallacy	a description is given of a 31-year old woman named Linda who is single, outspoken, and very bright. She is described as a former philosophy major who is deeply concerned with issues of discrimination and social justice. When asked which of two statements are more likely, fewer pick A: Linda is a bank teller, than B: Linda is a bank teller active in the feminist movement, even though A is more likely than B.	The conjunction of two correlated events is judged to be more likely than either of the events themselves. (Garfield, 1995)

It also important to note that both pre-service and in-service teachers often have many difficulties with the ideas of probability and statistics (Garfield & Ben-Zvi, 2007).

Specific research findings related to the “big ideas” of distribution, center, and spread

- Students confuse “case-value plots, where a bar or line represents an individual case, and histograms, where a bar represents multiple cases. These differences can cause confusion by students, leading them to try to describe shape, center and spread of case-value plots (see delMas, Garfield, & Ooms, 2005) or to think that bars in a histogram indicate the magnitude of single values (Bright & Friel, 1998)”.
- More time is needed for students to develop a sense of distribution as its own entity with characteristics (Wild, 2006; Konold & Higgins, 2003) (G & B, 2007)
- “school students frequently make poor choices in selecting measures of center to describe data sets (Zawojewski & Shaughnessy, 2000)." (Garfield & Ben-Zvi, 2007)

- “Students needs need a notion of distribution before they can sensibly choose between measures of center and perceive them as a ‘representatives” of a distribution.’” (Mokros and Russell (1995))
- "Current research on student statistical understanding of distribution recommends a shift of instructional focus from drawing various kinds of graphs and learning graphing skills to making sense of the data, for detecting and discovering patterns, for confirming or generating hypotheses, for noticing the unexpected, and for unlocking the stories in the data (Ben-Zvi, in press; Pfannkuch, 2006; Pfannkuch & Reading, 2006; Prodromou & Pratt, 2006; Reading & Reid, 2006; Watson, 2005)."
- "Understanding variability has both informal and formal aspects, moving from understanding that data vary (e.g., differences in data values) to understanding and interpreting formal measures of variability (e.g., range, interquartile range, and standard deviation). While students can learn how to compute formal measures of variability, they rarely understand what these summary statistics represent, either numerically or graphically, and do not understand their importance and connection to other statistical concepts." (Garfield & Ben-Zvi, 2007)

Broad Teaching recommendations:
Comprehensive literature reviews of teaching and learning statistics suggest providing ample opportunities to initially informal examination of sampling, center and spread slowly leading up to more formal notions (Garfield & Ben-Zvi, 2007). Examples could include build-up toward the Central Limit Theorem[3] that begins with carrying out simple experiments. Another example would be to utilize box plots to enable students to consider center, variance and distribution among different populations.

Research also recommends that students consider the topics of distribution, center and spread together rather than as separate topics.

Research has found that in most classrooms more time is spent on the procedural calculations of statistical measures and on generation of graphs. Research recommends more time spent on interpreting data and reading results from statistical tools.

Studies involving computer simulations and statistical software enabling students to visualize data appears to improve students’ understanding of random events and data analysis. (Garfield & delMas, 1991; Simon, Aktinson & Shevokas, 1976; Weissglass & Cummings, 1991; Weissglass & Cummings, 1991; Rubin, Rosebery & Bruce, 1988).

Activity-based courses and use of small groups appear to help students overcome some misconceptions of probability (Shaughnessy, 1977) and enhance student learning of statistics concepts (Jones, 1991). (G, 1995) Several studies support cooperative learning in teaching statistics (Garfield & Ben-zvi, 2007). Students working in pairs and groups during the class scored higher in exams and were more likely to stay in the course when compared to students in lecture course (Garfield & Ben-Zvi, 2007).

Research on variability suggests beginning with “informal ideas” and “gradually moving to more formal ideas and measures” with the use of carefully designed activities recognizing “the different ‘faces’ of

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[3] CLT roughly states: The distribution of an average will tend to be Normal as the sample size increases, regardless of the distribution from which the average is taken OR if we take the mean of the samples and plot the frequencies of their mean, we get a normal distribution. See (http://www.statisticalengineering.com/central_limit_theorem_(summary).html)

variability, such as overall spread in a data set, variability between two data sets, variability as measurement error, etc." (Garfield & Ben-Zvi, 2007). "It is also suggested to focus students' attention on the nature and sources of variability of data in different contexts, such as variability in a particular data set, outcomes of random experiments, and sampling (Shaughnessy, Watson, Moritz, & Reading, 1999; Gould, 2004).

Research supports connecting the concepts of shape, center, and spread early in students' experiences noting, "the notion of an average understood as a central tendency is inseparable from the notion of spread" (Konold and Pollatsek, 2002). (Garfield & Ben-Zvi, 2007).

"There is a growing consensus to emphasize general distributional features such as shape, center and spread and the *connections among them* in students' early experiences with data.

Students' ideas about the likelihood of samples (related to the representativeness heuristic) are improved by having them make predictions before gathering data to solve probability problems, then comparing the experimental results to their original predictions (Shaughnessy, 1977; delMas & Bart, 1987; and Garfield & delMas, 1989).

When students are tested and provided feedback on their misconceptions, followed by corrective activities (where students are encouraged to explain solutions, guess answers be-fore computing them, and look back at their answers to determine if they make sense), this 'corrective-feedback' strategy appears to help students overcome their misconceptions (e.g., believing that means have the same properties as simple numbers) (Mevarech, 1983).

Common Core

Common Core Mathematics Practices, Domains, and Combined Clusters (of Standards)

These modules will focus on a wide set of Common Core Practices and Standards. Below is a list of the practices and the Standard domains to be focused on:

1. Make sense of complex problems and persevere in solving them.
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

Statistics within the Common Core
Descriptive Statistics:

- Develop understanding of statistical variability. 6.SP
- Summarize, represent, and describe distributions on measurement variables and categorical variables. 6.SP, S-ID
- Investigate chance processes and develop, use, and evaluate probability models. 7.SP
- Investigate patterns of association in bivariate data. 8.SP

Inferential Statistics:

- Making Inferences and Justifying Conclusions S-IC
  Understand and evaluate random processes underlying statistical experiments S-IC
  Make inferences and justify conclusions from sample surveys, experiments, and observational studies 7.SP, S-IC
  Draw informal comparative inferences about two populations. 7.SP
  Interpret linear models (Use technology to obtain linear models); Distinguish between correlation and causation S-ID

  Using Probability to Make Decisions S-MD
  Calculate expected values and use them to solve problems S-MD
  Use probability to evaluate outcomes of decisions S-MD

Conditional Probability and the Rules of Probability S-CP
Understand independence and conditional probability and use them to interpret data S-CP
Use the rules of probability to compute probabilities of compound events in a uniform probability model S-CP

Critical References

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