

Folding fractals

1. Fold a long strip of paper in half, right over left. Unfold it. We'll call this \vee to represent the direction of the fold.
2. Fold the strip back in half, like it just was, and fold it in half again, right over left again. Unfold it. It should now look like $\vee\vee\wedge$, by which we mean that the first two folds are "up" and the last fold is "down".
3. Make an organized table of the folding patterns with three, four, five, and six folds, along with the one and two fold patterns you already have. Can you find a rule that helps you predict what the seven fold pattern will look like? Perhaps it would be easier to keep track of if you used some other symbols for the up and down folds – ones that don't look so similar, like 1 and 0 perhaps.
4. Try writing one sequence of folds at half size under the one before it. Does this suggest a different rule than the one you just found?
5. Explain why each of the two rules you found in the previous parts should work.
6. Arrange your folded strip carefully so that each fold is a right angle. So after one fold, when viewed on edge, it looks like \lfloor . What does it look like after two folds? Three folds? And so on. Eventually it should touch itself but not cross itself.
7. Starting with the digit 1 written down, and your finger on the 1, repeat the following rules, each time writing some digits at the end of your list and then moving your finger one digit over:
If your finger is on a 1, write a 32 on the end.
If your finger is on a 2, write a 42 on the end.
If your finger is on a 3, write a 31 on the end.
If your finger is on a 4, write a 41 on the end.

After you've written for a while, replace each odd digit with a \vee and each even digit with a \wedge . What do you observe? Why does this work?