

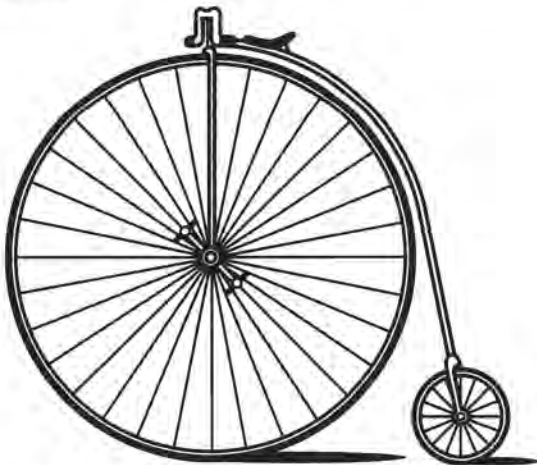
Gearing Up

In this lesson you will learn about the mathematics of gears. This will help you understand the decisions people have to make when they buy or design bicycles.

RELAY RACE

- How far does a bicycle travel for every revolution of the wheel for each wheel diameter below?
 - 20 in.
 - 27 in.
 - 50 in.
 - 64 in.

Old-fashioned bicycles had huge front wheels. Most of these high-wheelers, as they were called, had a 50-inch front wheel and a 17-inch rear wheel, but some of the makers got carried away and built front wheels as high 64 inches! The pedals were in the center of the front wheels.



- Why did bicycle makers make such big wheels?

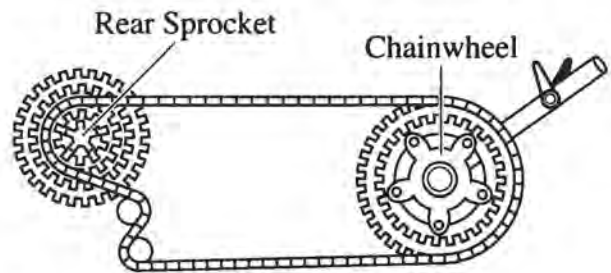
Highwheelers had two drawbacks. First, the rider had to work very hard to get started, and most of these bicycles had to be pushed or dragged up hills. Second, their height made

them a dangerous and impractical means of transportation. The rider had to jump down from the seat when the bicycle stopped, hoping to land feet-first.

The invention of gears on bicycles was a key development. Gears allowed the rider to travel longer distances for each turn of the pedals, without requiring such big wheels.

HOW GEARS WORK

Example: A bicycle has a chainwheel having 45 teeth and a rear sprocket having 15 teeth.



Each time the chain passes over one tooth on the chainwheel, it also passes over one tooth on the rear sprocket. Therefore, the rear sprocket will go through three revolutions for every one revolution of the chainwheel.

- Explain why riding a 27-inch bicycle having these gears would be like riding an 81-inch bicycle in terms of the distance covered in one turn of the pedals.

Definition: The *gear ratio* is the ratio of the number of teeth on the chainwheel to the number of teeth on the rear sprocket.

- If the gear ratio is 2.5, how many turns does the rear wheel make for each turn of the pedals?

A ten-speed bicycle has two chainwheels and five rear sprockets. Each combination of chainwheel and sprocket is a different gear.

5. Make a table to show how the gear ratio changes as a function of the number of teeth on the gears of a ten-speed bicycle, with two chainwheels having 40 and 54 teeth, and five rear sprockets having 14, 17, 22, 28, and 34 teeth.

	14	17	22	28	34
40	—	—	—	—	—
54	—	—	—	—	—

6. **Generalization** Write the gear ratio (g) as a function of the number of teeth on the chainwheel (c) and the number of teeth on the rear sprocket (r).

RELAY RACE

Definition: The *gear* is the gear ratio multiplied by the diameter of the rear wheel. It gives the diameter of the wheel that would travel the same distance in one revolution of the pedals. (The unit of gear is inches, but it is usually omitted.)

Example: The gear ratio is $40/20$, or 2, when using a chainwheel having 40 teeth and a rear sprocket having 20 teeth. On a bicycle having 26-inch wheels, the gear would be 2×26 , or 52. This means that each turn of the pedals when the bicycle is in this gear would move the bike a distance equivalent to one turn of a 52-inch wheel.

7. If the gear is 52, how far would the bike travel with each turn of the pedals?


8. Generalization

- Write the gear (G) as a function of the number of teeth on the chainwheel (c), the number of teeth on the rear sprocket (r), and the size of the wheel (w).
- If the gear is G , how far would the bike travel with every turn of the pedals?
- Write a formula that gives the distance (d) that the bike would travel with each turn of the pedals as a function of c , r , and w .

RELAY RACE

Definition: The *cadence* is the pace of pedaling.

A good cadence to maintain is 65 to 85 pedal revolutions per minute. Better cyclists like to maintain a cadence of 90 turns per minute.

9. Julio's ten-speed bike has wheels 27 inches in diameter. Its gears were described in problem 5. At a cadence of 90 pedal revolutions per minute, how fast, in miles per hour, would Julio be going in the highest gear? (Hint: Find a conversion factor to get directly from pedal revolutions per minute to miles per hour.)
10.  If Julio knows his cadence, find a way for him to calculate his speed mentally in miles per hour when riding in the highest gear.

DESIGN A BIKE

11. **Project** Design a bicycle. First describe the future owner of the bicycle and his or her needs. Will the rider be climbing steep hills? Be racing? Choose a size for the wheel, and the number of teeth for the gears of a 10-, 15-, or 18-speed bicycle. The following information may be helpful. Describe how each gear would most likely be used.

Wheel diameters

24, 26, and 27 inches are common.

Teeth on the chainwheel

24 to 58

Teeth on the rear sprocket

12 to 38

Sample Gears

- Very low gear, for climbing steep hills and for easy starts: 33
- Medium gear, for general use: 54
- Very high gear, for going downhill fast, and for racing: 100

Progression

Some cyclists like an approximately geometric progression of gears, because the common ratio makes the change feel the same from one gear to the next.

