**Maximum Area of a Rectangle with Fixed Perimeter**

Date: 03/03/2004 at 11:02:13

From: John

Subject: area of a rectangle

If I am given a specific length of fence, such as 128 feet, how can I

calculate the maximum amount of square footage that I can enclose in a

rectangle using the fence?

At first I thought that given a specific fixed length of fence any

rectangle would yield the same square footage. But that's not true.

I believe that there must be an equation to figure this out.

I used guess and check and ultimately found that a square of 32 x 32

feet yielded the most square footage. Then altering it slightly to

make a rectangle was the best I could come up with. But it was all

guessing.

Date: 03/03/2004 at 11:51:33

From: Doctor Douglas

Subject: Re: area of a rectangle

Hi John,

Thanks for writing to the Math Forum.

Actually, the 32 ft x 32 ft square gives the maximum area of any

rectangle of fixed perimeter 128 ft.

Here's how to convince yourself of this. Any rectangle must have a

length L and a width W. For the perimeter to be 128, 2L + 2W = 128,

or L + W = 64. The area of this rectangle is L\*W. We want to pick

the L and W such that we get the maximum area L\*W.

Now, let's rewrite our equations as follows. Let the length be

L = 32 + x and the width be W = 64 - L = 64 - (32 + x) = 32 - x. Our

task now is to see what value of x gives the best (maximal) area:

 area = L\*W = (32 + x)(32 - x)

 = 1024 + 32x - 32x - x^2

 = 1024 - x^2

Now, x^2 is a positive number for any x not equal to zero, so that we

will always reduce our area from 1024 sq ft, which is achieved only at

x = 0, or L = W = 32 ft.

This argument doesn't use calculus, which is a more powerful method to

compute maximum and minimum quantities and where they occur.

- Doctor Douglas, The Math Forum

 <http://mathforum.org/dr.math/>

Date: 03/03/2004 at 13:02:13

From: John

Subject: area of a rectangle

Thank you for your quick response. I was going to point out that the

32 by 32 arrangement is a square and I am looking for the rectangle of

largest area, but then I remembered that a square is in fact also a

rectangle! Thanks for all your help on this!

**Regular and Non-regular Polygon Areas**

Date: 03/10/99 at 17:10:37

From: Robert Davies

Subject: Regular Polygons

I am trying to work out the proof that regular polygons give the

maximum area but as of yet have not succeeded.

Please help!

Date: 03/11/99 at 11:55:35

From: Doctor Peterson

Subject: Re: Regular Polygons

The first approach that comes to mind is to take any non-regular

polygon and show that you can find a larger polygon with the same

perimeter. Then the largest polygon with that perimeter must be

regular.

Suppose that there are three consecutive vertices A, B, and C in a

polygon such that AB and BC have different lengths. See if you can find

a point B' where AB' and B'C are the same length, but their sum is the

same as AB + BC. Then show that the area of triangle AB'C will be

larger than that of ABC. If you replace B with B' in the polygon, its

perimeter stays the same but the area is larger.

 B'

 B +

 + / \

 / \ / \

 / / \ \

 / / \ \

 / / \ \

 / / \ \

 / / \ \

 A // \\ C

 + - - - - - - - - - - - - - - - - - +

 | \

 | \

 ... ...

This will show that the largest polygon has to have all sides the same,

since a polygon whose sides are not the same is never the largest.

You will also have to show that the angles in the largest polygon with

a given perimeter are all the same. Try a method similar to what we

just did for the sides. (I have not taken the time to work that part

out.)

- Doctor Peterson, The Math Forum

 <http://mathforum.org/dr.math/>

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| **Understanding Rectangle Area and Perimeter**Date: 11/08/2002 at 16:50:32From: Mark; Subject: AreaDear Dr.Math,I am a sophmore in high school and we are reviewing area and perimeter. My teacher gave us a problem and told us to prove it true or false. I know that the answer is false, but I need help understanding and explaining why it's wrong.Here is the problem: We have a given square and we see that if we increase the perimeter, area increases as well in our new rectangle. Is that always true? Give a thorough explanation and clear examples.Given Square: ------  3ft | |  | |  | | ------  3ft P = 12ft and A = 9 sq.ftNew Rectangle: --------- | | 3ft | | | | --------- 4ft  P = 14ft and A = 12 sq.ftI do see that the theory that "as perimeter increases, the area as well increases" is not always true, even though it works for the given problem. My example is that if our new rectangle looks like this, then our perimeter increased but our area decreased, so the theory is wrong or at least not always right. ---------------------- 1ft | | ----------------------  6ft P = 14ft and A = 6ftI know how to do the math and figure out perimeter and area, but I don't know how to explain why it doesn't work if the one side of the rectangle is one, and why it did work for the given problem.Please help,Sincerely,MarkDate: 11/09/2002 at 17:22:49From: Doctor RickSubject: Re: AreaHi, Mark.You are correct that a rectangle with perimeter greater than 12 (the area of the given square) does not necessarily have an area greater than 9 (the area of the given square). When you are told to disprove a statement, all you need to do is to provide a counterexample - a case that satisfies the premises (rectangle with perimeter greater than 12) and does not fit the conclusion (area greater than 9). You have done this, so you have a thorough explanation with a clear example.You don't need to explain "why" the statement is not true in any deeper sense than "because here is an example in which it is not true." But I understand your desire to understand it on a deeper level.The fact is that if you require that the perimeter of a rectangle be THE SAME as that of the square, the area of the rectangle will ALWAYS be LESS than that of the square (unless the rectangle is the square itself). In other words, for a given perimeter, the rectangle that has the largest area is a square.Let's consider a rectangle with length L and width W. Its perimeter is 2(L+W) and its area is LW. If we require that the perimeter be some particular value P, then we can find what the width must be when we know the length of the rectangle: 2(L+W) = P L + W = P/2 W = P/2 - LThen the area will be LW = L(P/2 - L) A = (P/2)L - L^2Thus the area of the rectangle is a quadratic function of the length L. Its graph is a parabola that opens downward, and the greatest area occurs at the vertex of the parabola. You can prove that the value of L at the vertex is P/4, so that the width is also P/4 and the rectangle with greatest area is a square.If you have further questions about this, please feel free to ask me. "Why" questions can lead in lots of different directions, and only you can tell me when we have hit on an explanation of the type that will satisfy you. Of course, a true mathematician will never be completely satisfied, there are always more directions to explore.- Doctor Rick, The Math Forum <http://mathforum.org/dr.math/>  |
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###  Why a Square Maximizes Area/Perimeter

Date: 07/24/2002 at 10:21:44

From: Elizabeth

Subject: area and perimeter

I need to come up with a rectangle with an area greater than 16 ft.

squared, and a perimeter of 16 ft.

Date: 07/24/2002 at 12:27:34

From: Doctor Ian

Subject: Re: area and perimeter

Hi Elizabeth,

Has someone led you to believe that this would be possible? A square

with perimeter 16 ft has an area of 16 ft^2:

 4

 +---+

 | | 4

 +---+

Suppose we keep the same perimeter, but use another shape. To do

this, we would have to reduce the width by some amount, x, and

increase the length by the same amount:

 4-x

 +--+

 | |

 | | 4+x

 +--+

Now the perimeter is still 16 feet, but the area is

 A = (4+x)(4-x)

 = 4^2 - x^2

 = 16 - x^2

square feet. Which is to say, \_any\_ change we make, from a square to

a non-square rectangle, will result in a smaller area.

Does that make sense? - Doctor Ian, The Math Forum

### Rectangles: Area and Perimeter the Same

Date: 4/15/96 at 22:20:22

From: Anonymous

Subject: Problem solving

There are two different rectangles whose sides are integers and

whose area and perimeter are the same. Find the dimensions.

Date: 4/16/96 at 10:17:11

From: Doctor Sebastien

Subject: Re: Problem solving

Let x and y be the sides of the rectangle.

The area is xy and the perimeter is 2(x+y)

xy = 2x + 2y

xy - 2x - 2y = 0

x(y-2) = 2y

x = 2y/(y-2)

Now find values of x and y such that x>0, y>0 and x and y are

integers.

When y = 3, x = 6.

When y = 4, x = 4

-Doctor Sebastien, The Math Forum

### Equal Area and Perimeter: Rectangles

Date: 09/09/2001 at 13:56:35

From: Jessica

Subject: Area and perimeter

Hi!

There are only two rectangles whose area is exactly the same as their

perimeter if the dimensions of each are whole numbers. What are the

dimensions?

Thanks,

Jessica

Date: 09/10/2001 at 12:46:10

From: Doctor Ian

Subject: Re: Area and perimeter

Hi Jessica,

Let's look at a rectangle for a moment:

 a

 +------------+

 | |

 | | b

 +------------+

The area will be a\*b; the perimeter will be 2a + 2b. So we want to

find pairs of numbers (a,b) such that

 a\*b = 2a + 2b

More specifically, we want to find pairs of whole numbers for which

this equation is true. One simple way to attack a problem like this

is to start making a table:

 a b a\*b 2a+2b

 --- --- --- -----

 0 0 0 0

 0 1 0 2

 1 0 0 2

So we've found one solution already. A 0 by 0 rectangle has an area

equal to its perimeter.

(This seems kind of strange, I know, but it fits the definition. And

note that the problem explicitly says that the numbers must be 'whole

numbers', not 'counting numbers'. The only difference between

counting numbers and whole numbers is that the whole numbers include

zero. So whenever you see a problem that talks about 'whole numbers',

that's a tip that zero might turn out to be important.)

Now, the tough part about constructing a table like this is to make

sure that you cover all the possibilities. How can we guarantee that

we get all the possible (a,b) pairs? What if we miss one, and that

turns out to be the answer? We could keep looking forever.

Well, here is a trick for ordering these pairs:

 |

 10 11 12 13

 |

 5 6 7 14

 |

 2 3 8 15

 |

 1----4----9---16---

The coordinates of the points, in order, are:

 1: (0,0)

 2; (0,1)

 3: (1,1)

 4: (1,0)

 5: (0,2)

 6: (1,2)

 7: (2,2)

 8: (2,1)

 9: (2,0)

 10: (0,3)

 11: (1,3)

and so on. Do you see why this will get \_every\_ pair of whole numbers?

Of course, it will get some of them twice (for example, (2,1) and

(1,2) actually describe the same rectangle), which creates some extra

work. (I included both (0,1) and (1,0) in the table above before I

realized this. I didn't need to work out both of them.)

If we want to be lazy (which in math is often a good thing), we can

just look at the points above the diagonal:

 |

 7 8 9 10

 |

 4 5 6

 |

 2 3

 |

 1------------------

(Do you see why this works?)

If we want to be even lazier, we can note that since

 a\*b = 2a + 2b

 = 2(a + b)

the product a\*b must be an even number. This is a break, because it

means that we can ignore any (a,b) pair in which both a and b are odd.

So we can forget about possibilities like (1,3), or (3,5), since there

is no way to multiply two odd numbers to get an even number.

Anyway, now you can make a table, which will let you find the other

answer to the problem.

Now, when making a table like this, there are two possibilities:

 1. You'll run across the answer very quickly. If that happens,

 then using a table will turn out to have been a good idea.

 2. You won't run across the answer very quickly. If that happens,

 there are two more possibilities.

 a. You'll notice some pattern in the table that will

 tell you where the answer has to be, without your

 having to construct all the table entries in between.

 If that happens, then using a table will turn out to

 have been a good idea.

 b. You won't notice any pattern like that. In that

 case, you'll want to start looking around for another

 way to solve the problem.

Fortunately, in this case, you should come across the answer pretty

quickly, if you use both of the shortcuts that I mentioned.

Does this help? Write back if you'd like to talk about this some

more, or if you have any other questions.

- Doctor Ian, The Math Forum

 <http://mathforum.org/dr.math/>