Math Matters, Chapin and Johnson, pgs. 88-90

Divisibility Rules

Objective: Review and apply the rules of divisibility.

Review the rules regarding divisibility by 2, 3, 4, 5, 6, 8, 9, and 10. (There is also a divisibility test for 7 but it is neither simple nor efficient and thus is not included; checking to see whether a number is divisible by 7 is best done with a calculator.)

Divisibility by 2: The number is even
Divisibility by 3. The sum of the digits is divisible by 3.
Divisibility by 4: Either the last two digits are 00 or they form a number that is divisible by 4.
Divisibility by 5: The ones digit of the number is either 0 or 5.
Divisibility by 6: The number is even and the sum of the digits is divisible by 3.
Divisibility by 8: Either the last three digits are 000 or they form a number that is divisible by 8.
Divisibility by 9: The sum of the digits is divisible by 9.
Divisibility by 10: The ones digit of the number is 0.

Determine which of these numbers (2, 3, 4, 5, 6, 8, 9, and 10) divides evenly into 234,567,012. Then develop a divisibility rule for 15 and for 18.

Things to Think About

What is the number 234,567,012 divisible by? It is divisible by 2 because it is an even number. It is divisible by 3 because the sum of the digits is 30 ($2 + 3 + 4 + 5 + 6 + 7 + 0 + 1 + 2 = 30$) and 30 is divisible by 3. It is divisible by 4 because the last two digits form the number 12, and 12 is divisible by 4. It is divisible by 6 because it is both an even number and divisible by 3. It is not divisible by 5, 8, 9 or 10.

The fact that 234,567,012 is not divisible by 8 when it is divisible by 2 and 4 leads us to consider how the divisibility of a number relates to the factors of that number. In order for a number (n) to be divisible by another number (a), it must possess all of the prime factors of that number. The prime factorization of 8 is $2 \times 2 \times 2$ or 2^3; the number 8 has only one prime factor, 2, but it is multiplied by itself three times! For 234,567,012 to be divisible by 8 it must have 2^3 in its prime factorization; 234,567,012 has 2×2 in its prime factorization because it is divisible by 4, but it does not have the additional 2 as a prime factor to make it divisible by 8.

Todetermine whether a number is divisible by 15, consider the prime factors of 15—3 and 5. Applying divisibility rules for both 3 and 5 will determine whether a number is divisible by 15. The same method can be used for 18: apply the divisibility rules for 2 and for 9 to any number to determine whether it is di-visible by 18. Why can't you use the divisibility rules for 3 and 6 to determine whether a number is divisible by 18? Again, it is related to factors. The prime factorization of 18 is $2 \times 3 \times 3$ or 2×32 Thus, for a number to be divisible by 18, it must possess all of those factors. When you use the divisibility rule for 6, you are checking to see whether a number contains the factors 2 and 3. When you use the divisibility rule for 3, you are again checking to see whether the number has 3 as a prime factor (it does—you determined that with the 6 rule), but you aren't checking to see whether the number has two 3s (3×3) as factors. Another example may help clarify this idea: we know that 24 is divisible by 3 and 24 is divisible by 6, but 24 is not divisible by 18 even though it is divisible by 3 and 6; why not? The rule for divisibility by 3 and the rule for divisibility by 6 both check to see whether 3 is a factor of 24 (it is), but the rules do not check to see whether 9 (3×3) is a factor of 24 (it isn't).

What if you want to create a rule for divisibility by 12? Which divisibility rules should you use? Did you pick 2 and 6 or 3 and 4? Only if we use the rules for 3 and 4 can we be sure that a number is divisible by 12. If we use the rules for 2 and 6, we don't learn if the number has two 2s (2×2) as factors.

Think about the examples discussed: a rule for 15 uses rules for 3 and 5, a rule for 18 uses rules for 2 and 9, and a rule for 12 uses rules for 3 and 4. Why do these rules work, whereas the combination of rules for 3 and 6 (for 18) and for 2 and 6 (for 12) do not work? If divisibility rules in combination are to work, the numbers can't have any factors in common. Numbers that have no prime factors in common are said to be relatively prime. Thus, 3 and 4 are relatively prime; the only factor common to both is 1.