Exploring Algebraic Thinking in a Math Teachers' Circle

NYC Community of Adult Math Instructors (CAMI) &
Adult Numeracy Network

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CUNY Adult Literacy & HSE Program
New York City

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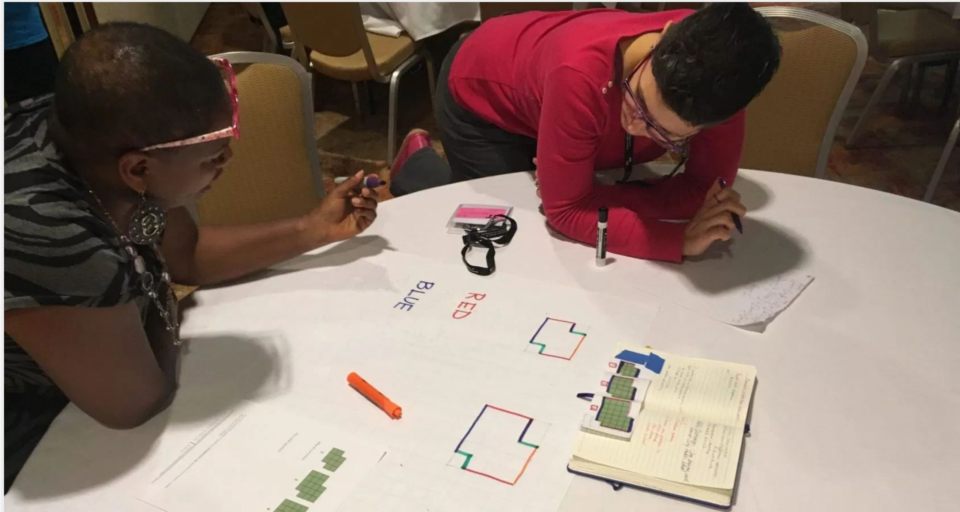
#ANNmath



Agenda

- Welcome
- Problem-Posing
- Problem-Solving
- Presenting Solutions
- Connections to Teaching

The NYC Community of Adult Math Instructors (CAMI)

- A group of teachers from different programs across NYC who get together once a month to do math and talk about teaching
- A problem-solving approach to teaching math
- One problem per meeting
- Open-ended, non-routine problems
  - No obvious solution method
  - Multiple strategies

In CAMI, we believe in the power of adult education teachers doing math together, making connections between our own learning and our teaching.

“No matter how kindly, clearly, patiently, or slowly teachers explain, they cannot make students understand. Understanding takes place in the students' minds as they connect new information with previously developed ideas, and teaching through problem solving is a powerful way to promote this kind of thinking.”

Diana Lambdin, 2003*

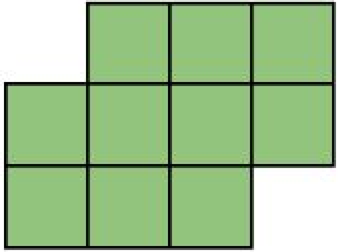
from Teaching Mathematics through Problem Solving, by Diana Lambdin

Problem Posing

Launch

We are going to show you a figure.

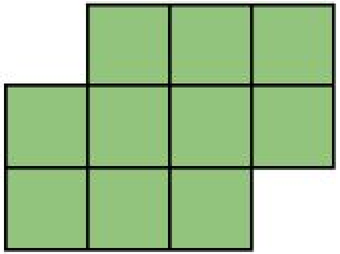
- What do you see?
- Try to keep a visual image of it in your mind.



2

What did you see?

What did you notice?



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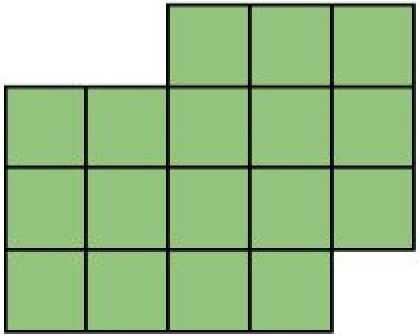

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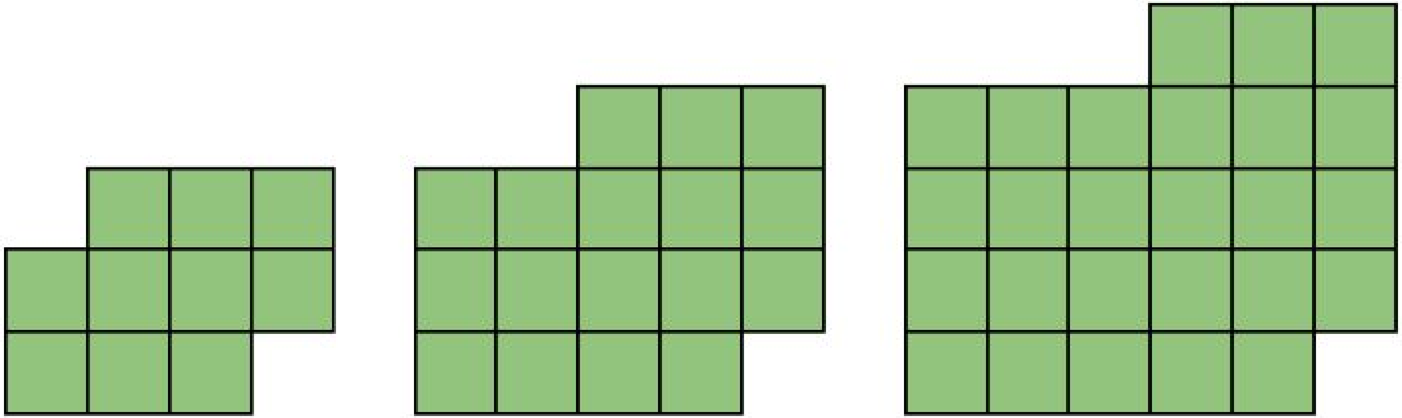
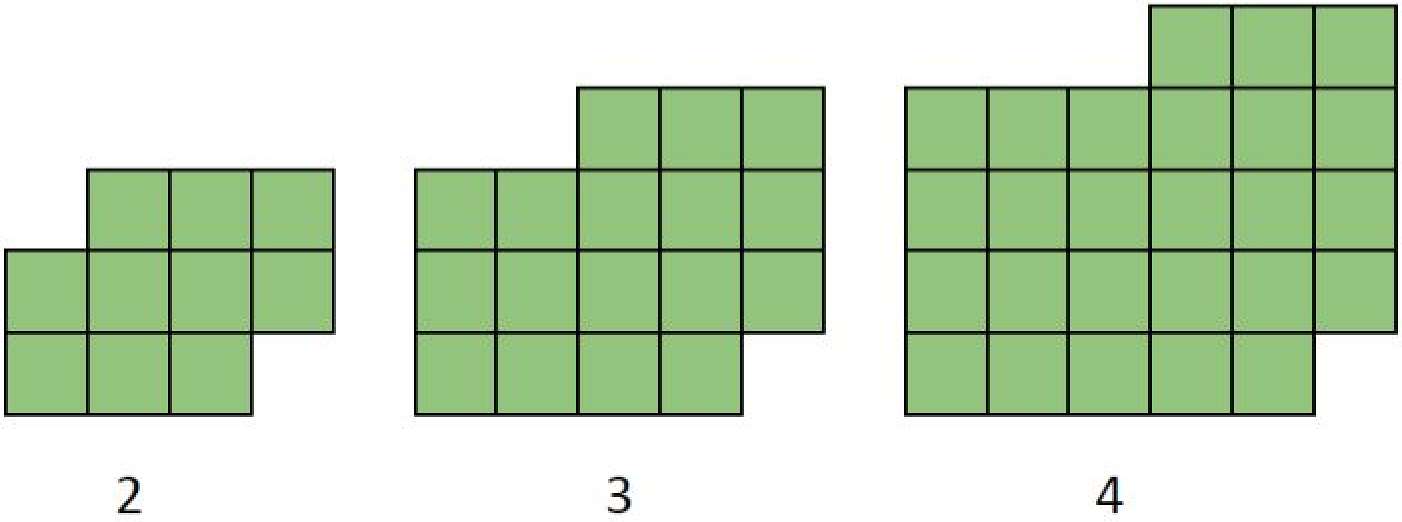
What did you see?

What did you notice?

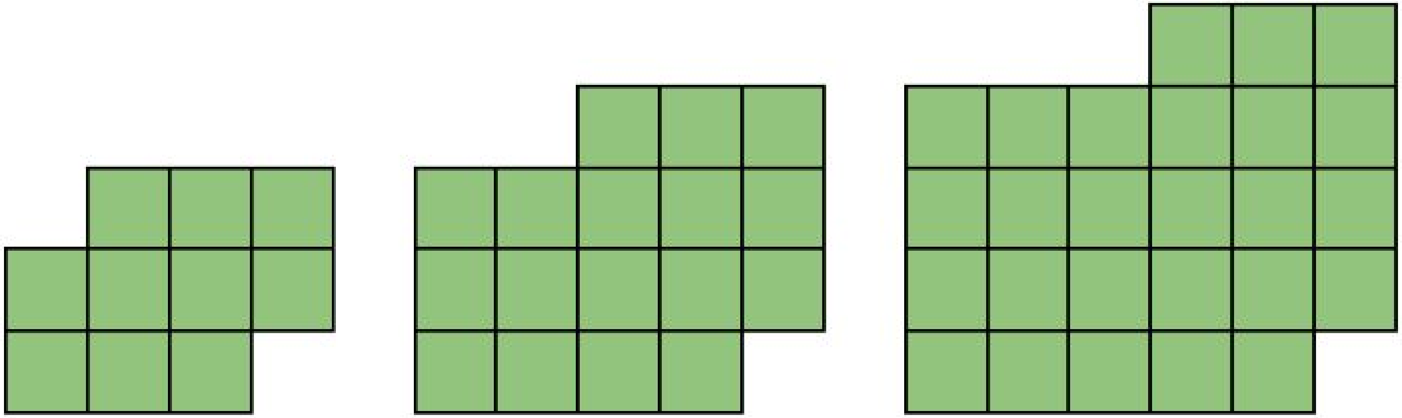


2 3 4

What did you see?



What Do You Notice?



2 3 4

As the figure number changes,

____________________ also changes.

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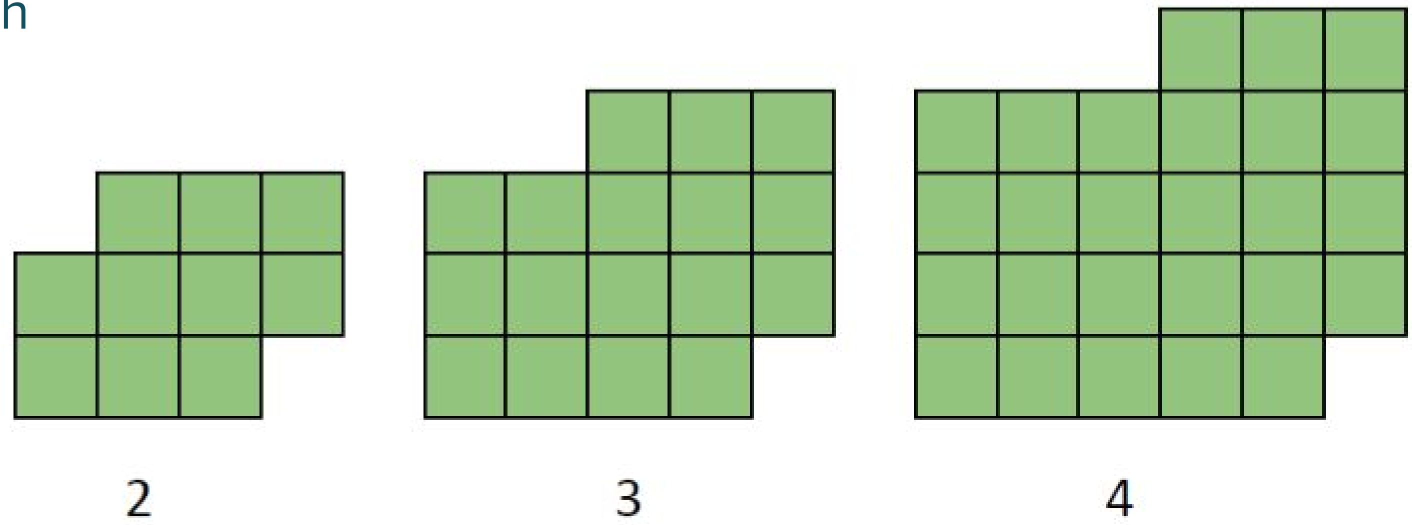
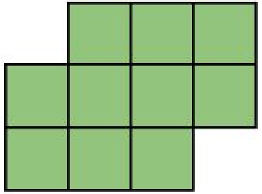


Draw the next figure.



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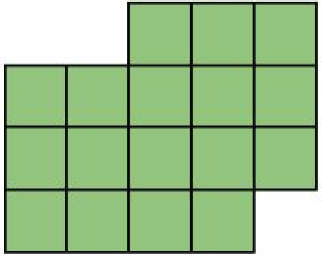

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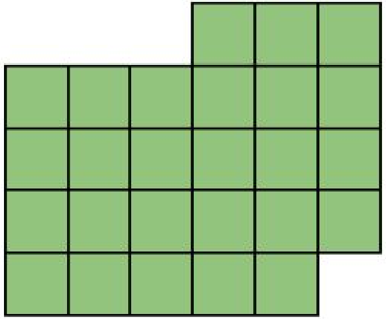

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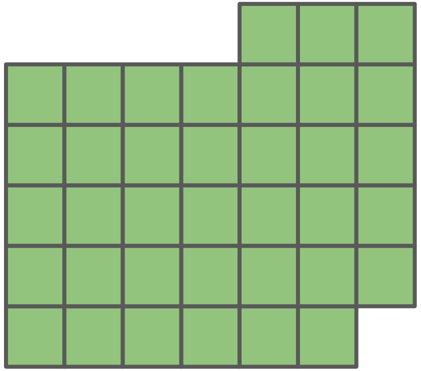

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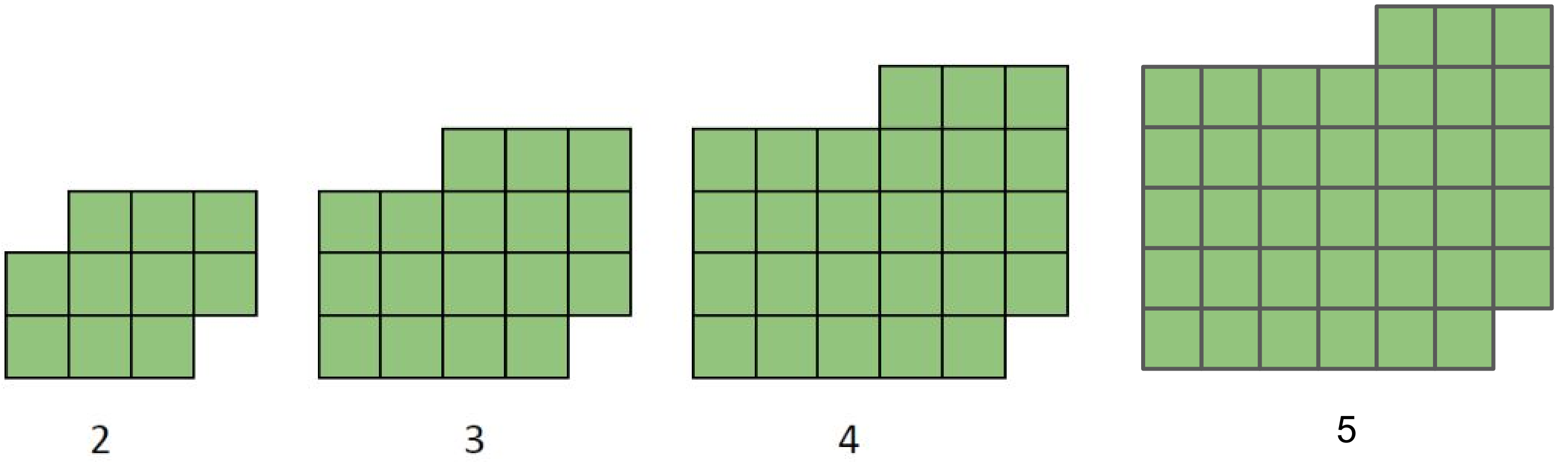
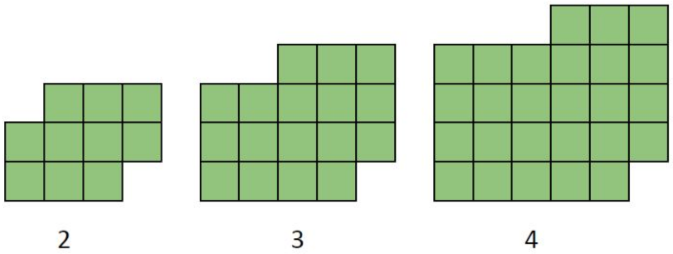
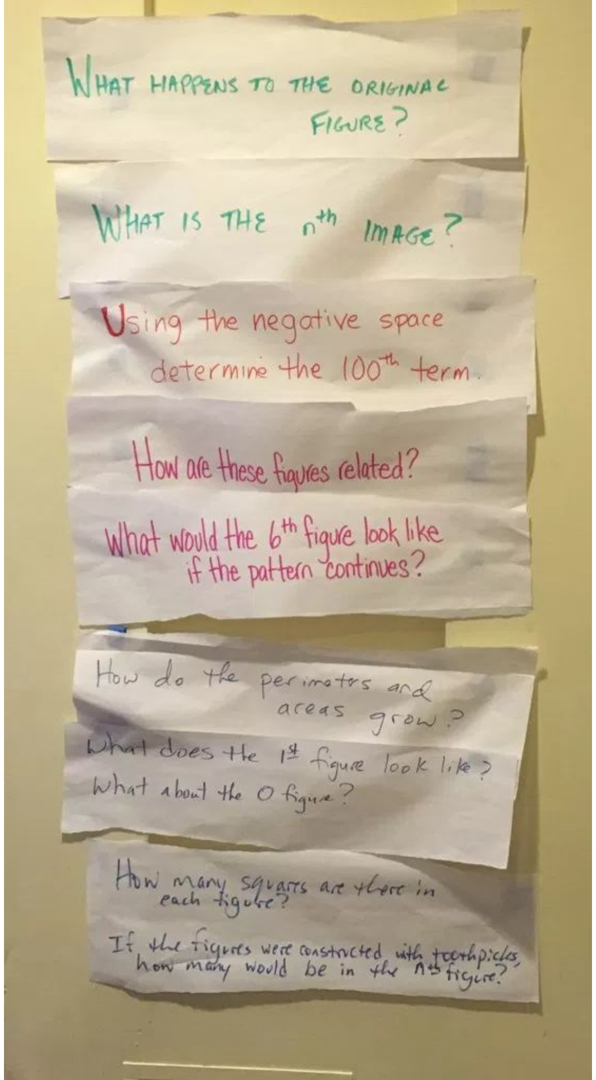


Pose some questions.

Letting Students Build the Problem

- What would figure 1 look like?
- What would the next figure look like if the pattern continued?
- What does figure 100 look like?
- How could I calculate the number of squares for any figure number?
- How could I describe how to draw the 19th figure?
- What is constant in all three figures?





Problem Solving

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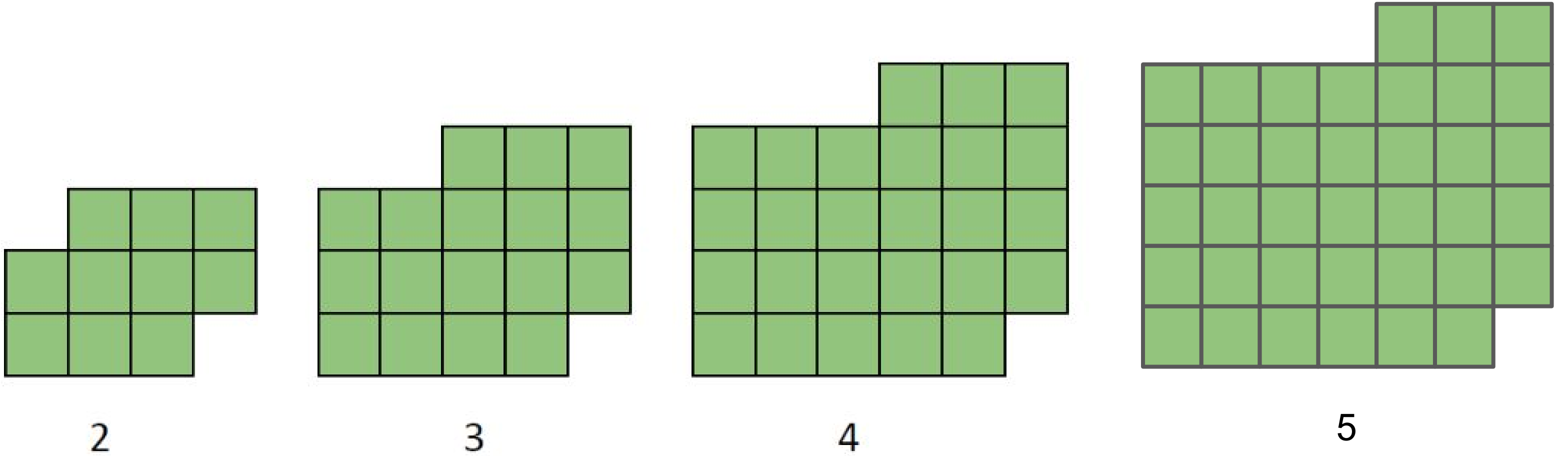
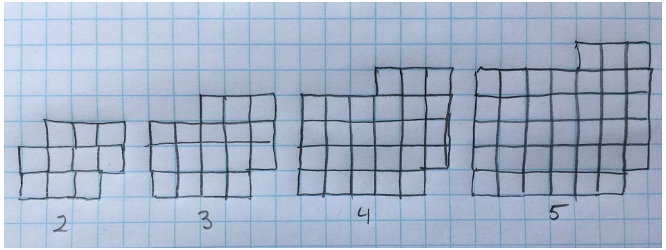
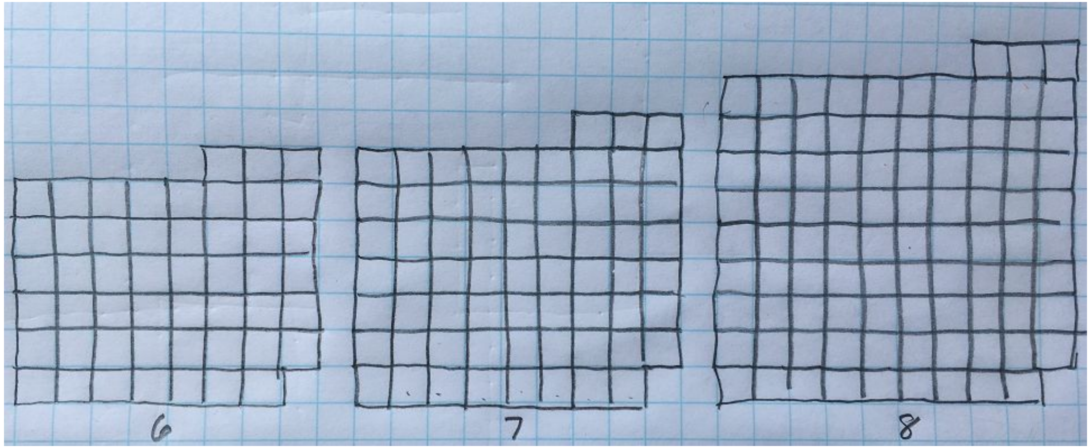


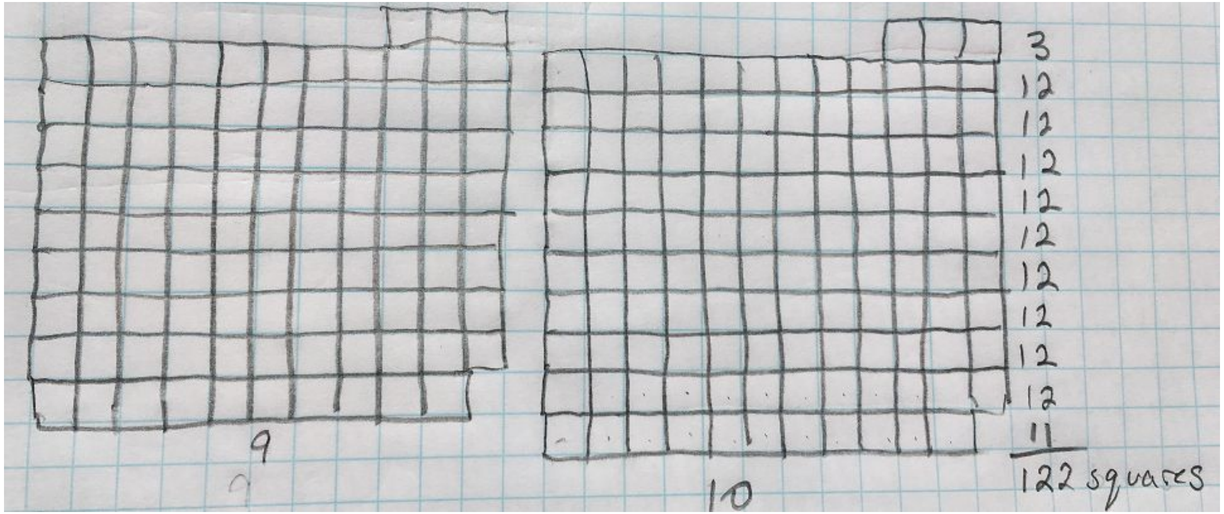
How many squares would be in the 10th figure?

Presenting Solution Methods



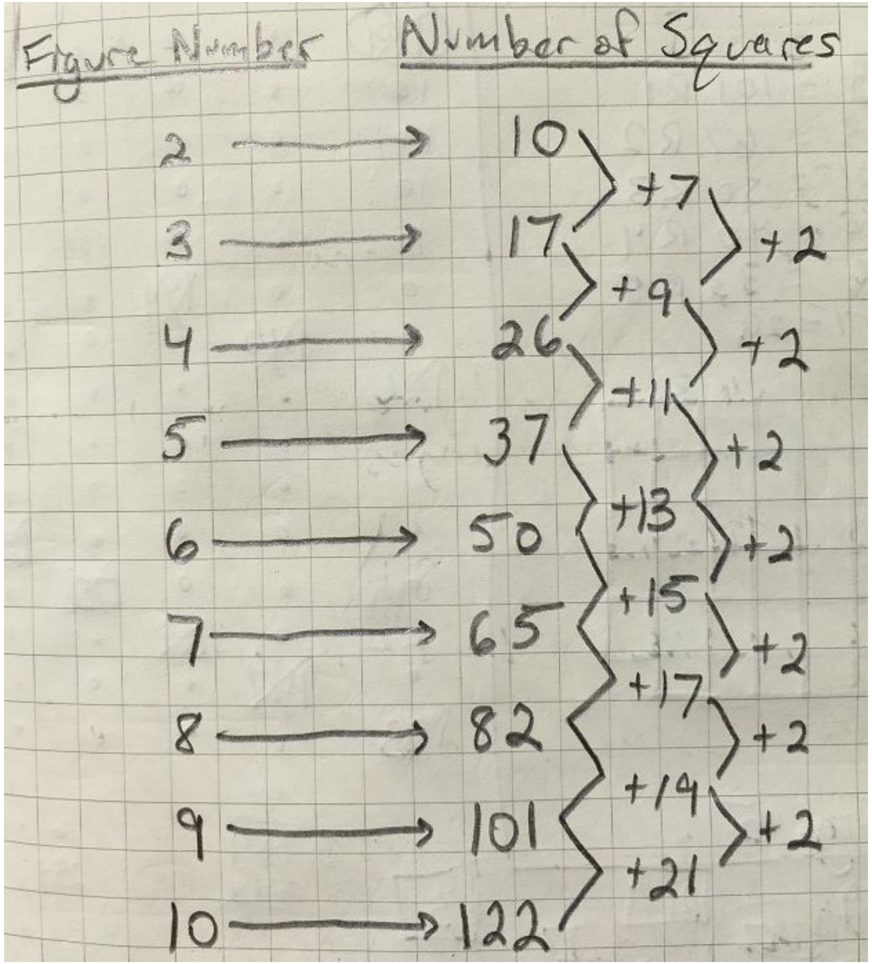


Drawing the next 5 figures

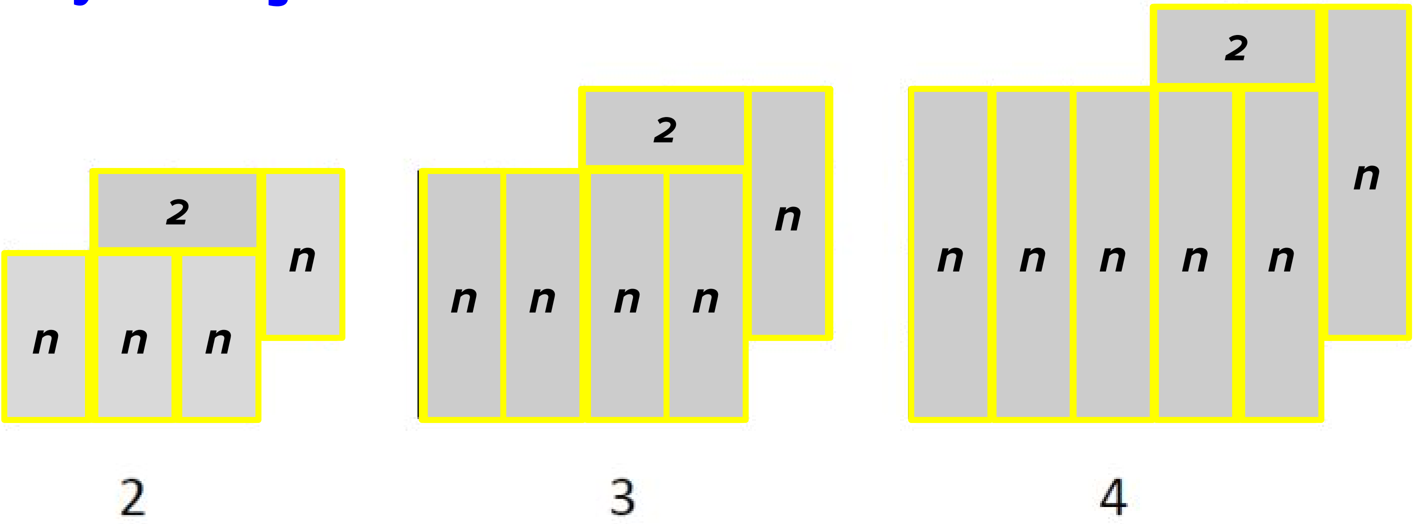


Make a table.

Look for patterns in the numbers.



Mark's Way of Seeing

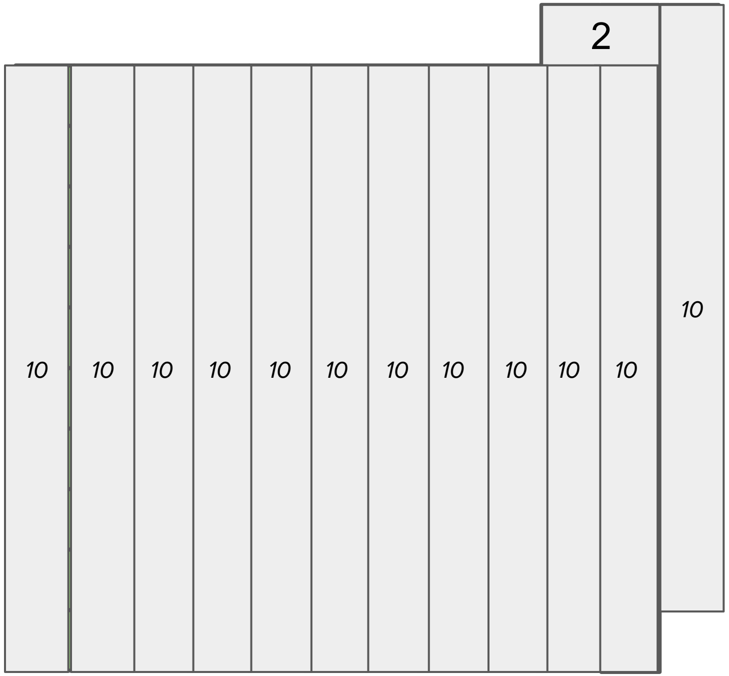


4*n*+2 **5*n*+2** **6*n*+2**

The number of squares in each figure can be found by multiplying the figure number by two more than the figure number and then adding two.

$$(n+2)n + 2$$

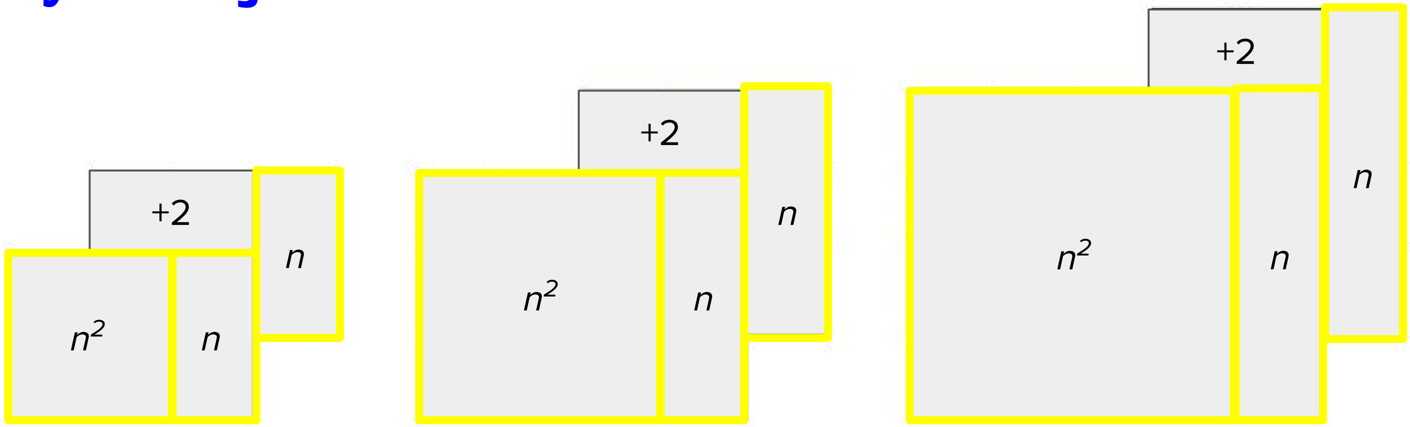
Mark's Way of Seeing the 10th Figure



$(n+2)n + 2$

$(10 + 2)(10) + 2 = 122$

Eric's Way of Seeing



2 — $n^2 + 2n + 2$

3 — $n^2 + 2n + 2$

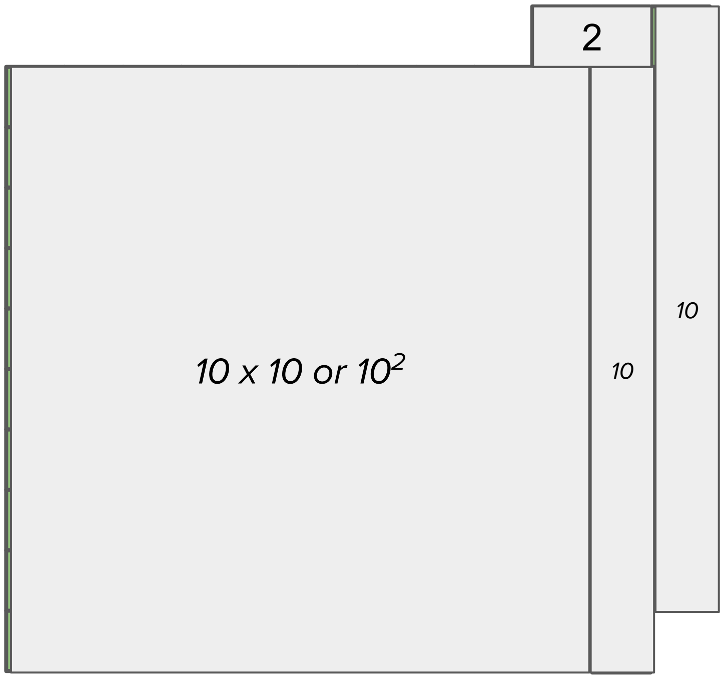
4 — $n^2 + 2n + 2$

The number of squares in each figure can be found by squaring the figure number, adding two more groups of the figure number and then adding two.

or

$$n^2 + 2n + 2$$

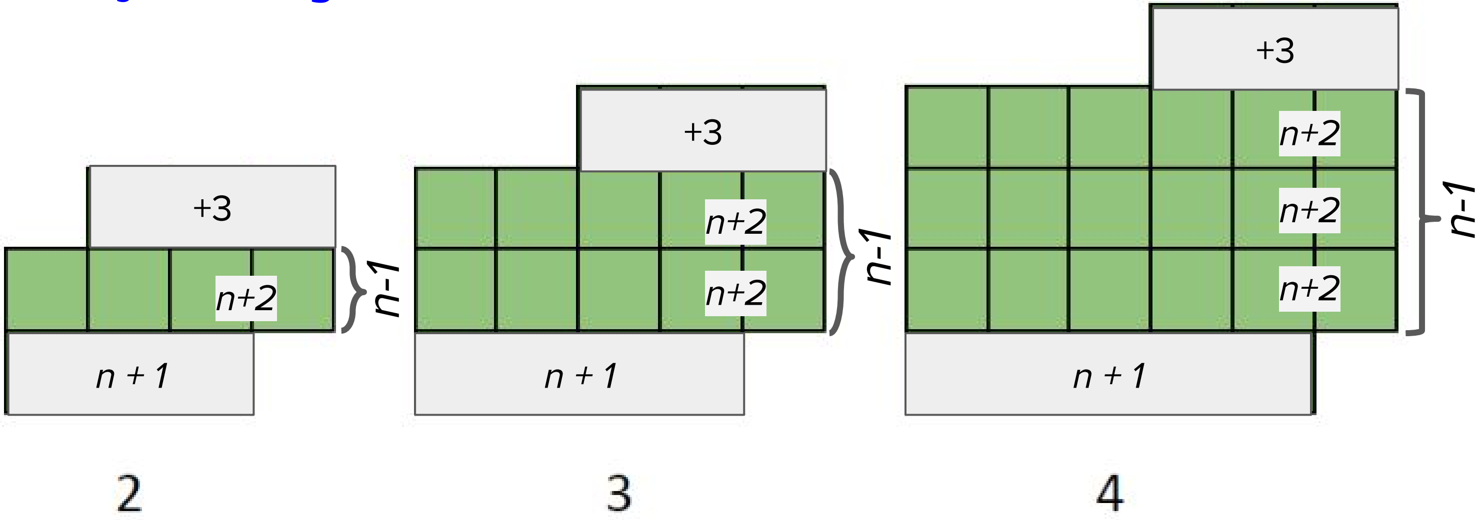
Eric's Way of Seeing the 10th Figure



$$n^2 + 2n + 2$$

$$10^2 + 2(10) + 2 = 122$$

Solange's Way of Seeing



The top row is always three squares.

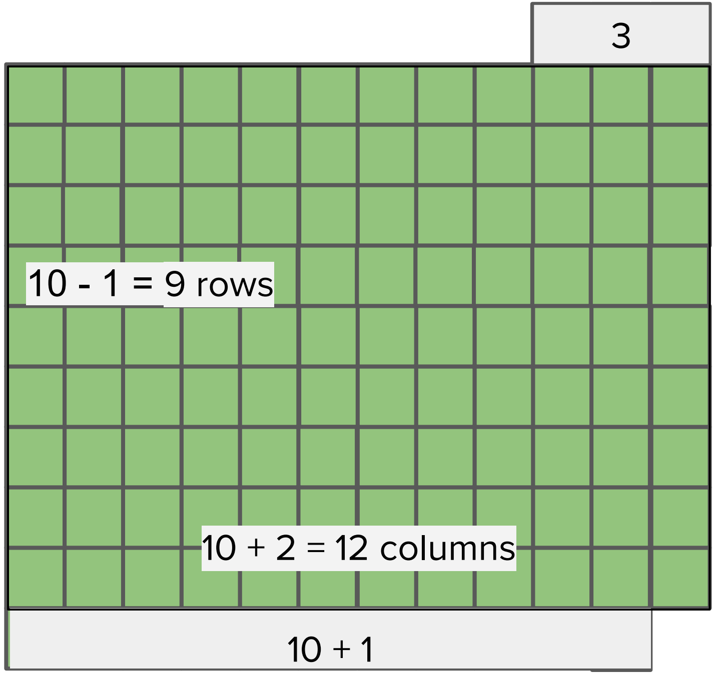
The bottom row is always the figure number plus 1.

The number of squares in each row in between is always the figure number plus 2.

The number of rows in between is always the figure number minus 1.

$$3 + (n + 1) + (n + 2)(n - 1)$$

Solange's Way of Seeing the 10th Figure



$$3 + (n + 1) + (n + 2)(n - 1)$$

(12 x 9 = 108)

3 + 11 + 108 = 122

Connections to Our Teaching

Questions teachers can ask students to advance their mathematical thinking (a sample)

- Ways to get unstuck: What do you know about the problem? What question am I working on? What are the special conditions to pay attention to?
- Can your organize your data in a table?
- What stays the same in each figure?
- Can you predict what the 20th figure would look like? What about the 200th?
- If you know there are 258 squares in a figure, what figure number is it? How do you know?
- Where does the 3 come from in the rule?
- Can you use *x* (another student's) strategy to find the 11th figure?
- Can you explain *x* (another student's) strategy in your own words?
- Which strategy is most efficient? Why do you think so?

Connecting Visual Patterns to Algebraic Thinking

Algebra is the generalization of arithmetic. - Marilyn Burns

- Generalizing solution methods
- Organizing data; creating tables of values
- Input/Output
- Functions
- Variable as an *unknown* vs. representing *any* number
- Equivalent equations
- Multiple representations of function: graph, words/story, picture, table of values

Resources

- *Problem Posing and Problem Solving in a Math Teacher's Circle,* COABE Journal Spring 2017
- CUNY HSE Math Curriculum Framework: Problem-Solving in Functions & Algebra: **CollectEdNY.org/FrameworkPosts**
- Fawn Nguyen's **visualpatterns.org**
- CAMI's Website: **NYCCAMI.org**
- Adult Numeracy Network (ANN): **adultnumeracynetwork.org**
- COABE Adult Ed Repository: **adultedresource.coabe.org**

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