NRP Sums of Consecutive Numbers[1]

NRP (2 Days): Section 1-Lesson #1

Objectives/ Driving Questions:

The "Sums of Consecutive Numbers" problem is an ideal problem to use for starting students into the practice of non-routine problem solving. The accessibility factor is high because the beginning of the explorations just requires being able to add consecutive numbers. All students will be able to find certain patterns emerging. The challenge in this activity lies in getting students to express the findings they will have. This could be an opportunity to introduce the greatest integer function as an extension.

Common Core Mathematical Practices:

Recognizing patterns (MP. 8)
Writing rules from patterns (MP. 2, MP. 8)

Common Core Standards:

Sample key learning tasks / assessment tasks:

Now use your rule to predict whether each of the following numbers can be made with two consecutive numbers, three consecutive numbers, four consecutive numbers, etc. and what those numbers for summing would be. Explain why you made the predictions you did. Then check them to see if they work.

a. 45 b. 57 c. 62

Lesson Segments and possible time frames		Prerequisite Skills
1.5 days		Adding numbers
Intro to consecutive numbers	5 min	
Table work completion	30 min	
Recording findings	15 min	
Problem 8	20 min	
Sharing methods	20 min	

Day 1: Have students work on completing the table, seeking patterns and expressing patterns. Additional time could be on having students share some "solutions" to 8a, b, and c.

[1] Adapted from Driscoll's *Fostering Algebraic Thinking, 1999.*

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Day 2: Create some problems similar to those in number 8 for students to use their rules with. Challenge students to confirm whether their expressions are valid.

Teacher set-up, notes, and guiding questions

Notes

Considerations

Letting students first work independently on finding patterns will enable students working at different paces to still discover patterns of their own. However after students have had time to express the patterns they have noticed, getting them to discuss findings and check the validity of their peer's expressions is useful. **Class discussion could easily begin with problems from number 8 or other similar problems. This would ask students to use their findings with larger numbers. This could also be a place to have students test their "rules" to see if others can interpret and use them correctly.**

Struggles to anticipate/Accessibility issues

- What about zero? Response: Try it out, no rule against it
- What about negative integers? Response: Try it out, no rule against it

Scaffolding options

- This problem can be edited to reduce the exploration to 2, 3, 4, and 5 consecutive numbers rather than the current version below which extends up to seven consecutives.
- Pairing students with others likely to work at similar paces could become a way to have students work jointly early in the lesson.
- Give students some time to look over a multiplication table (or particular rows/columns of a multiplication table). Do student yet know methods of determining when a number is divisible by 3? Are odd and even clear terms?

Scaffolding cautions

- Students need to have time to be genuinely and independently seeking out patterns.

Warm-up task options

- If I give you a number, how do you find a number half its size? Tell the story of Gauss as a mischievous student in class… teacher asked him to add the numbers from 1 to 100. What was his short cut? Try it out on the sum of 1 though 10.
- For each of the following numbers list all of their factors: a) 45, b) 46, c) 47, d) 48, e) 49

Notes: There is some redundancy in questions 2 through 7. Questions 2 and 3 ask students to state the patterns they see, to make predictions from these patterns, and to write a rule for their pattern. Some students with just these two questions might go into detail with results for several cases. Questions 4 through 7 guide students more particularly to look at certain cases looking for patterns in case they had not seen them when asked the more general questions in 2 and 3.

Sums of Consecutive Numbers

Instructions

- Individually, or in groups of two or three, work through the following math activity.
- As you work, think about the strategies you use to solve the problem.

Math Activity

7 + 8 = 15   2 + 3 + 4 = 9   4 + 5 + 6 + 7 = 22

The expressions above are examples of the sums of consecutive numbers. The number 15 is shown as the sum of two consecutive numbers. The number 9 is shown as the sum of three consecutive numbers. The number 22 is shown as the sum of four consecutive numbers. In this activity, you will explore how to make different numbers with sums of consecutive numbers.

Number	Two Numbers	Three Numbers	Four Numbers	Five Numbers	Six Numbers	Seven Numbers
1						
2						
3	1+2					
4						
5						
6		1+2+3				
7						
8						
9						
10						
11						
12						
13						
14						
15	7+8					
16						

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Number	Two Numbers	Three Numbers	Four Numbers	Five Numbers	Six Numbers	Seven Numbers
17						
18						
19						
20						
21						
22			4+5+6+7			
23						
24						
25						
26						
27						

Number	Two Numbers	Three Numbers	Four Numbers	Five Numbers	Six Numbers	Seven Numbers
28						
29						
30						
31						
32						
33						
34						
35						

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2. What patterns do you see in the chart? (You might look for numbers that can be made by a particular length of sum, or how many different-length sums can make particular numbers, or which numbers can't be made by consecutive sums. Use your imagination)

3. What predictions about sums of consecutive numbers can you make from these patterns? Write a rule if you can. How general is your rule?

4. If you haven't already, look just at the numbers that can be made by a sum of three consecutive numbers. What patterns do you see in those numbers? How can you predict whether a number can be made by a sum of three consecutive numbers? Which consecutive numbers will add up to these numbers?

5. Now do the same thing for sums of five consecutive numbers. Can you predict which numbers can be made by such sums and what consecutive numbers will make them?

6. Make a prediction about sums of other odd numbers of consecutive numbers. Test your prediction for an example or two. Why does your prediction work?

7. Does this rule work for sums of an even number of consecutive numbers? Why or why not? If not, how might you modify the rule to make it work?

8. Now use your rule to predict whether each of the following numbers can be made with two consecutive numbers, three consecutive numbers, four consecutive numbers, etc. and what those numbers for summing would be. Explain why you made the predictions you did. Then check them to see if they work.

    a. 45   b. 57   c. 62   d. 75   e. 80