problems did you encounter? Was your method effective, or can you think of a better way to have worked?" For a discussion of groups' problem-solving strategies, ask: "What strategy did your group use? Did any group use a different strategy?" Allow enough time for all groups to present their methods.

When presenting solutions, groups should state their findings and, whenever possible, display their work. Ask: "How did you decide if your findings make sense? How can you check your solution?"

Generalizing a solution involves extending it to other situations not necessarily dependent on the specific limitations of the problem. The following questions are effective in eliciting generalizations: "Are there patterns or relationships you can see from your solution? Can you think of another problem you've solved that this reminds you of? How are the two problems alike and different?" It's also helpful to give the students an altered version of the problem and ask how their solution or methods of solution would change.

Before they summarize, all the students should have ended their explorations and should have prepared for a class discussion. Having children come together in a gathering place helps them focus. Other classroom hints: Have students speak one at a time; direct groups to choose a spokesperson to report to the entire class; encourage students to respond to each other's comments; record data on the board as it's presented.

Do not summarize any extension activities at this time unless all groups have had a chance to tackle them. They can be discussed later with the appropriate groups. Posing additional challenges at this time is one way to extend the problem for the more interested or able students.

About timing: At times, a class will complete all three stages within one math period. But keep in mind that teaching a lesson should not necessarily imply a one-day experience. Investigations can extend for two, three, or more days so that all groups have adequate time to explore and can contribute to the summarizing. Also, summarizing is extremely important and should not be skipped or shortened for lack of time. It's valuable for students to reflect on their learning, to hear from others, and to connect others' experience to their ideas. Don't let the clock push the curriculum; rather, stay attuned to the learning needs of the students.

A Sample Lesson: The Consecutive Sums Problem

A good beginning small-group investigation is *The Consecutive Sums Problem*. It reinforces a skill students have practiced—addition with sums to twenty-five—and extends this skill in a way that promotes cooperative small-group work. Following are directions for teaching this lesson.

Introducing

1. Present or review concepts.

Consecutive numbers are numbers that go in order, such as 1, 2, 3, 4 or 11, 12, 13. Each comes right after the other without skipping. Ask students for sequences of three or four consecutive numbers. Ask them to explain why 15, 16, 18, for example, are not consecutive.

2. Pose a part of the problem or a similar but smaller problem.

Ask: "Who can tell me a way to write the number nine as the sum of consecutive numbers?" The usual response is $4 + 5$. Record this on the chalkboard. Underneath this, write $9 = 2 + 3 + 4$. Ask them to verify if this is true. Tell the class that this shows that it's possible to write nine as the sum of consecutive numbers in at least two different ways. Try another example if you think it's needed.

3. Present the problem to be solved.

Ask the students, in their groups, to find all the ways to write the numbers from one to twenty-five as the sum of consecutive numbers. (For younger children, finding the sums for the numbers from one to fifteen may be sufficient.) Tell them that some of the numbers are impossible; challenge them to see if they can find the pattern of those numbers. Direct them to search for other patterns as well, such as how many different sums there are for different numbers. Provide groups with large paper for recording, and ask them to put their group label on their recording sheet (ace, 2, 3, etc.) as well as their group members' names. On their recording sheet, they should also write statements that describe the patterns they find.

4. Discuss the task.

Ask for questions. Review the guidelines for group work if you think that's necessary.

Exploring

1. Observe the interaction.

Notice work procedures. Some groups divide up the numbers so that one person does one to six, the next does seven through twelve, and so on. In other groups, individuals work on whichever numbers they choose and add their findings to a group chart. Some groups have one person do all the recording;

others share that job among the members. Recording formats differ as well. Some groups list the numbers from *1* to *25* and write the sums next to each. Other groups organize the numbers by how many different sums they found for each, so all those that could be written in only one way are in one column, those that could be written in two ways are in another, and so on.

2. Offer assistance when needed.

Sometimes a group will summon you to ask a procedural question, such as whether they should orient the paper the long way or the short way. Let them make those decisions for themselves; tell them they'll see later what other groups decided. Keep in mind that although it may seem like a minor decision to you, it isn't so for children. Organizing work on paper is a skill students need to acquire; group decisions can help them do so.

Groups sometimes make erroneous generalizations. For example, when they find it's impossible to write 2 and 4 as the sums of consecutive numbers, they may conclude that 6 would fit the pattern and also be impossible. In such a situation, confront them with a contradiction. Ask the group to consider 1 + 2 + 3. When they realize that the sum of those numbers is 6, leave them to rethink their hasty generalization.

You may notice a group bogged down in a way related not to the problem itself but to some procedural issue. For example, the group isn't keeping a group record. Join that group and ask: "How are you supposed to record your results?" (This way you find out if they understood your instructions about which paper to use.) Then ask: "Who will get the paper for your group?" (This may prompt someone to do so.) Or ask: "What do you need to do in order to get started?"

Sometimes, a group has written all the sums it can find and calls you over to announce it has finished. However, when you look at the recording, you see that the students haven't written any statements about patterns they've found. The usual response is that they can't think of any. Ask probing questions to kindle their thinking: "Can you see a pattern to the numbers that are impossible? How could you describe that pattern in a summary statement? What do you notice about all the numbers that had three possible sums? Which numbers had only one possible sum?" When the students have begun to consider some of your questions, leave them to write statements. Help them word one if you feel they need more support.

3. Provide an extension.

If a group has completed work to your satisfaction, offer a challenge: "Can you find a way to predict how many ways thirty-six can be written as the sum of consecutive numbers? Can you predict for any number?"

Summarizing

1. Have groups share their processes, both group procedures and strategies used.

Ask groups to report how they divided up the work. Ask if they thought their method was a good one or how they would change it if they had to extend this exploration to include the numbers up to fifty.

Discuss strategies the groups used. Usually, some took a number, such as twelve, and used the *guess-and-check* strategy to find sums that worked. Others worked the other way, using the *work backward* strategy, starting with the sums and then writing them under the appropriate total. Younger children may have used counters to help—the *use objects* strategy. Most groups use the *look for a pattern* and *make an organized list* strategies. The different methods are important to discuss so students become aware that there are a variety of ways to approach problems. Also, such discussions can provide valuable insights into students' approaches to learning.

2. Have groups present solutions.

If possible, have groups post their recording sheets for the others to see. Discuss any differences and similarities in solutions. Ask groups how they were sure they had found all the possible ways to write any particular number.

3. Generalize from the solutions.

Review the summary statements, being sure that when incorrect generalizations appear, you provide counterexamples to help students reconsider their ideas. Ask about the impossible numbers, drawing out as many different descriptions as possible. Encourage children to explain in their own words: "Start with one, and multiply by two." "They're doubles." "They're powers of two." Ask for the patterns for numbers that were written two ways, three ways, and so on. Ask older children to examine the primes and describe what they notice. Present the challenge of finding a way to predict how many ways any given number could be written.

Structuring Independent Work: The Menu

Another way to organize the classroom for instruction is to provide a menu of learning activities that can include problems, games, and investigations that students work on independently. All students do not have to be working on the same task at the same time.