**Conversation between Eric and Sam (with a question at the end)**

Want to help me prep for a math workshop? I want to do something called a 3-act math task with a group of teachers. Subject: poker and probability.

Act 1: show a video with a cliffhanger (main questions: will he get a royal flush? what are the chances?)
Act 2: people work to figure out math
Act 3: people present solutions, ideas to the group

Here's what I want to do. I didn't make it up.http://whenmathhappens.com/2015/03/09/royal-flush/

What is the poker game being played here?

> Texas Hold 'em basics:
>
> Each player gets two cards (then there's betting)
> Then there is a "flop" of 3 cards (more betting)
> Then a "turn" of one more card (bet)
> And finally the "river" of the final card.
>
> All 5 cards in the middle, face-up, are "common cards," so ANY player at the table can use any of those cards to "make their hand." A poker hand is ALWAYS 5 cards, so they can use 3 to 5 of the common cards and 0 to 2 of their own cards.
>
> The last card is the final common card – the "river." After the first two cards, no player gets individual cards at all.
>
> The burn cards are a kind of ritual/etiquette for "counting" the rounds of play/betting. There is actually a mistake here: in the video you see he burns 3 cards: one before the flop, one before the turn, and one before the river. That is CORRECT. But in act 3 they only show us 2 of the 3 burn cards.
>
> At any rate, in a real poker game the only known cards would be the two in your hand and the 4 on the board. So you'd have only one "out" – that is, the card needed to make your hand. Notice that this is an "inside" straight, as part of the royal flush. If the player held J, Q of spades and the board had 9, 10 of spades, then EITHER the 8 or the K would make a straight flush (not royal because it's not to the A), so there would be two outs. Here there is only 1; there can only ever be 1 for a royal flush.

Is this right?

12 cards dealt to players
4 cards dealt up
3 cards burnt
18 total cards dealt
34 cards left in the deck

royal flush = 1/34 (1 queen of spades left in the deck)
straight = 4/34 (4 queens left in the deck)
flush = 8/34 (8 spades left in the deck)

It seems clear that the two aces have the best chance of winning. There is only a 8/34 chance of a flush with 10, ace hand. That's about a 75% chance of the two aces being the high hand. Anyone has a 2/34 chance of drawing a king, but since they all have equal chance of that…

> Almost, but not quite. Let's go through the hands, first looking at what they have now (before the river), and then looking at what their odds are of making the high hand:
>
> A, A – two pair (ace and kings), high hand
> J, 5 – two pair (jacks and kings)
> 9, 5 – two pair (5s and kings)**
> 5, 6 – two pair (5s and kings)**
> A, 10 – pair of kings with an A kicker
> 8, 10 – pair of kings with a J kicker
>
> **These two hands are exactly tied because in each case the player would play the 5 from their hand and ALL FOUR cards from the board.
>
> OK, now what's the best result for each player?
>
> A, A – needs an A or K to make a full house
> J, 5 – needs a J to make a full house
> 9, 5 – needs a 5 to make a full house
> 5, 6 – needs a 5 to make a full house
> A, 10 – needs a Q of spades to make a royal flush; or a spade to make a flush; or a Q to make a straight
> 8, 10 – best card would be a 10 for two pair
>
> OK, what are the odds? This time we'll go from bottom to top in terms of percentage chance of making the hand:
>
> 8, 10 – nothing can happen to have him win, so 0%
> 5, 6 – All 4 5s are out, so there are no outs for this hand, - 0%
> 9, 5 - same as above, 0%
> J, 5 - The 5s are all gone, but there are two Js left, so 2/34 = 6%

A, 10 – 8 outs for flush; (including Q of S) 3 more outs for straight (minus Q of S), so 11/34 = 32% (8/34 chance of hitting a flush, but there is also a 4/34 chance of hitting a straight and a 1/34 chance of hitting a royal flush. Now, some of those overlap: so it's not really 13 outs. Of the 8 spades, one is the queen of spades, so we can't count it twice. And we also can't count that queen a third time as one of the 4 queens. I think that means we get 8 plus 3 = 11 outs. So overall there is a 11/34 (32%) chance that the A, 10 will get a hand that beats the *current* high hand. Moreover, if the A, 10 hits one of those outs, then by definition the A, A hand did not improve.)
A, A – one Ace, plus two possible Kings = 3/34 = 9%

In addition, the A, A has a 6% chance of losing to the J, 5, if a J comes. So overall there are 12 cards out there that will make the A, A the losing hand. That's a 35% chance of losing, so a 65% chance of winning.

Ah, so we can count the ways in which A, A loses and calculate the odds of that hand losing? There are 12 outs that include the ways A, 10 can win and 1 out that will make J, 5 the winner. 12/34 = 35% of A, A losing.

Yes, that's the logic I was using. I don't have independent verification that it's correct, but it makes sense to me.
Note: in a real game of poker the odds of the A, A winning would be HIGHER. Why? Because the A, A hand would bet BIG knowing he's ahead, and anybody holding J, 5 who is not a total idiot would fold. This would improve the A, A odds of winning to 68%.

Hmm… I now think all of these odds are incorrect in practice. In the video, we're shown everyone's cards, but if we were really playing, I would only know my two cards and the community cards. If I wanted to calculate my odds of a royal flush after the turn, I would have one card that would complete the royal flush and 46 possible cards (including the private cards and the burn cards that were dealt, since I don't know what they were). That would make the denominator of all the following 46, not 34. I guess we have to decide whether we're calculating the odds that a player would have from his/her position or the objective odds from the vantage point of knowing all the cards that have been dealt. If we choose the former, all the odds below change. I'm going to stick with the idea that we know all the cards even though that's not accurate for someone playing the game.

Yep, I think that's all correct. The example is about teaching odds, so they show you things you don't know. If the example were about playing poker, then the unknown cards are definitely 46.

Poker is often about using the FORCE of the odds, along with the power of betting to your advantage. If YOU know your odds of winning, and I place a bet that makes it irrational for you to call (because the amount of money you would win is not enough to make your bet rational, given your likelihood of winning), then you will fold. But when you fold, my chance of winning go up because I don't compete against you. Indeed, often this logic works to make everyone fold, so I win by definition. And then you see the opening for bluffing, since if I make you think I have better odds of winning (by betting as if I do), then you may fold and I will win…even though I had nothing.

And card players calculate odds like this four times every game?! After dealing, after the flop, after the turn, after the river? Are people already counting the number of outs early in the hand? Or are they more intuitive with it?

Lots of variation here. Some are really *counting* and some have a general sense of averages and possibilities. And some mainly play the other players and try not to think about odds.

I personally don't really count odds, but I count outs and I have a strong general sense of odds...

In any case, it is CRUCIAL to update the odds after each play of cards. For example: if I'm playing hold 'em and the flop comes down with 3 different cards (no pairs) then I know there CANNOT be a full house. So if I'm playing to a straight then I'm playing to "the nuts" (THE best hand possible). But if a card comes down on a turn that "pairs the board" (makes a pair there), then it's now possible to make a full house, so the flush I have or hope to have is not the best possible hand any longer. The odds change dramatically.

Moreover, the odds that matter are "implied odds," in most cases, and this involves the amount of the bet. It might be rational to call with only 3 outs of the pot has $20 in it and the bet is $1. But if the pot has $10 in it and the bet is $10, then it could be irrational to call. Implied odds are basically expected value calculations (you know those?) where it's about my expected return. If the odds say I have a 10% chance of winning, but I'd be winning $100 on a $5 bet, then I should call (I risk $5 with an expected value of $10 [i.e. 10% chance of winning $100]. But if the chance of winning is 10% and the bet is $40 to win $50, then it's irrational to call (I risk $40 with an expected value of $5 [i.e. 10% chance of winning $50]). And as I showed above, a good poker player will use implied odds against you: he'll bet big enough to make it irrational for you to call…so then you have to figure out if he has the cards he's suggesting he has!!!

All of this, by the way (NOT gambling) is why I like poker.

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Here's a question:

Using an expected value calculation, when would it be rational for A, 10 to call a bet at this point in the game (all the cards have been dealt except for the river)? When would it be irrational to call?